Which coefficients in a two-group measurement model (CFA) does Mplus constrain across groups by default?

Authored by

stats writer

July 1, 2024

The two-group measurement model in Mplus is a statistical method used to analyze and compare data from two distinct groups. In this model, there are certain coefficients that are constrained across both groups by default. These coefficients refer to the relationships between the observed variables and the underlying latent constructs. By constraining these coefficients across groups, it ensures that the measurement model is equivalent for both groups, allowing for a fair and accurate comparison between them. This default constraint in Mplus helps to reduce potential biases and improves the validity of the results obtained from the two-group measurement model.

Which coefficients in a two-group measurement model (CFA) does Mplus constrain across groups by default? | Mplus FAQ

Note that this page contains a description of the defaults, other specifications are possible.

For a model with all continuous variables

The short answer

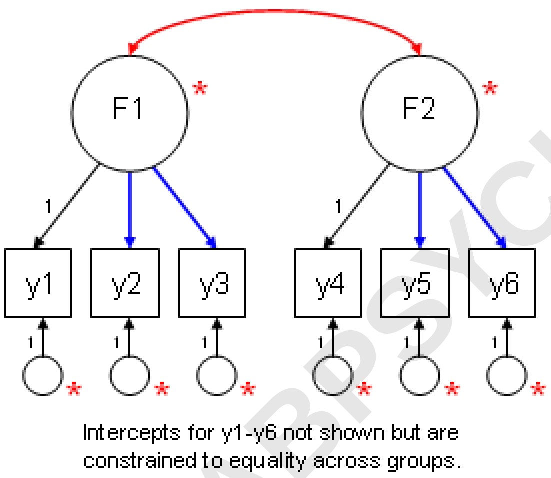
Directly below are lists of which coefficients are fixed and free across groups in a measurement only model. Further below is an example and more detailed explanation.

The following parameters are fixed to equality across groups:

The following parameters are allowed to be different across groups:

The following coefficients are fixed to an arbitrary (but customary) value for identification of the model:

The figure below graphically shows which parameters are fixed, and which are free. Paths that are constrained to equality across groups are shown in blue. Paths and other coefficients that are allowed to differ across groups are shown in red. Paths in black with values next to them are all constrained to one.



An example with explanation

Below is a two-group model (the groups are males and females), with three latent variables (x1, x2 and x3). All of the observed variables are continuous.

Data:

File is D:datamydata.dat;

Variable:

Names are a1 a2 a3 b1 b2 b3 b4 d1 d2 d3 female;

grouping is female (0 = male 1 = female);

Missing are all (-9999) ;

Analysis:

Type = general ;

Model:

x1 by a1 a2 a3;

x2 by b1 b2 b3 b4;

x3 by d1 d2 d3;

We have edited the output to put the parameter estimates and standard errors for each group next to each other (Mplus prints the output sequentially by group). You can view or download the full, unedited, Mplus output by clicking here. To make it easier to follow along we will examine the output in sections, starting with the factor loadings (indicated with the key word BY).

By default Mplus sets the factor loading for the first manifest variable listed to one in order to identify the

model. (This is arbitrary, other values, or manifest variables, and even other methods, can be used to identify the model.) Comparing subsequent factor loadings for each of the latent variables (factors) we can see that both the coefficients, and their standard error are the same. In other words, Mplus has constrained them to equality for males and females by default.

Males Females

Estimate S.E. Estimate S.E.

X1 BY

A1 1.000 0.000 1.000 0.000

A2 0.934 0.023 0.934 0.023

A3 0.771 0.027 0.771 0.027

X2 BY

B1 1.000 0.000 1.000 0.000

B2 1.100 0.068 1.100 0.068

B3 0.075 0.020 0.075 0.020

B4 0.029 0.009 0.029 0.009

X3 BY

D1 1.000 0.000 1.000 0.000

D2 1.014 0.081 1.014 0.081

D3 0.523 0.047 0.523 0.047

Next we will compare the covariances among the latent variables. Here, the estimates are very different. For example, the covariance of X1 with X2 is -0.237 for males and 0.17 for females. Clearly they are not constrained to equality.

Males Females
Estimate S.E. Estimate S.E.
X2 WITH
X1 -0.237 0.222 0.170 0.111

X3 WITH
X1 -0.004 0.038 0.024 0.025
X2 0.119 0.034 0.054 0.019

Looking at the means of the latent variables you probably notice right away that for males, all three latent variables have a mean of zero. In contrast, the estimated means for females are different from zero (although not significantly so in two of the three cases). This is related to identification of the model. Because there is no unique solution for the means of the latent

variables, the model estimates the difference between the means of the latent variables by group, (rather than values of the means of the latent variables for each group). In order to estimate the difference in means between groups, the mean for one of the groups is fixed to some arbitrary value, typically zero.

Males Females
Estimate S.E. Estimate S.E.
Means
X1 0.000 0.000 0.084 0.166
X2 0.000 0.000 -0.201 0.131
X3 0.000 0.000 0.010 0.027

Below we see that the intercepts for the observed variables are constrained to equality across groups.

Males Females
Estimate S.E. Estimate S.E.
Intercepts
A1 4.202 0.142 4.202 0.142
A2 4.120 0.135 4.120 0.135
A3 4.348 0.116 4.348 0.116
B1 0.630 0.118 0.630 0.118

B2 0.685 0.128 0.685 0.128

B3 0.172 0.027 0.172 0.027

B4 0.080 0.013 0.080 0.013

D1 0.084 0.020 0.084 0.020

D2 0.069 0.023 0.069 0.023

D3 0.067 0.015 0.067 0.015

Below we see that both the variances of the latent variables and the residual variances of the manifest (observed) variables are allowed to be different across groups.

Males Females

Estimate S.E. Estimate S.E.

Variances

X1 3.481 0.388 3.082 0.219

X2 2.365 0.302 1.511 0.151

X3 0.065 0.009 0.076 0.010

Residual Variances

A1 0.185 0.084 0.027 0.053

A2 0.524 0.091 0.626 0.063

A3 1.094 0.127 1.075 0.081

B1 0.321 0.144 0.362 0.102

B2 0.070 0.172 0.215 0.120

B3 0.496 0.052 0.359 0.025

B4 0.066 0.007 0.107 0.007

D1 0.006 0.005 0.109 0.010

D2 0.034 0.006 0.009 0.007

D3 0.046 0.005 0.072 0.005

For a model with categorical observed variables

The short answer

Directly below are lists of which coefficients are fixed and free across groups. Further below is an example and more detailed explanation.

The following parameters are fixed to equality across groups:

The following parameters are allowed to be different across groups:

The following coefficients are fixed to an arbitrary (but customary) value for identification of the model:

An example with explanation

Below is a two-group model (the groups are males and

females), with two latent variables

(x1 and x2).

In this model, all of the observed variables are dichotomous.

Data:

File is D:datamydata.dat ;

Variable:

Names are

a1d a2d a3d b1d b2d b3d female;

Missing are all (-9999) ;

categorical are a1d a2d a3d b1d b2d b3d ;

grouping is female (0 = male 1 = female);

Analysis:

Type = general ;

Model:

x1 by a1d a2d a3d;

x2 by b1d b2d b3d;

We have edited the output to put the parameter estimates and standard errors for each group next to each other (Mplus prints the output sequentially by group). You can view or download the full, unedited Mplus output by clicking here. To make it easier to

follow along we will examine the output in sections, starting with the factor loadings (indicated with the key word BY). By default Mplus sets the factor loading for the first manifest variable listed to one in order to identify the model. (This is arbitrary, other loadings, manifest variables, and even other methods, can be used to identify the model.) Comparing subsequent factor loadings for each of the latent variables (factors) we can see that both the coefficients, and their standard error are the same. In other words, Mplus has constrained them to equality for males and females by default. (It is very unlikely that the model would produce identical estimates unless the coefficients were constrained to equality across groups.)

Male Female
Estimate S.E. Estimate S.E.
X1 BY
A1D 1.000 0.000 1.000 0.000
A2D 0.757 0.255 0.757 0.255
A3D 1.345 0.601 1.345 0.601

X2 BY

B1D 1.000 0.000 1.000 0.000
B2D 0.857 0.083 0.857 0.083
B3D 0.860 0.086 0.860 0.086

Next we will compare the covariances among the latent variables. Here, the estimates are very different in the two groups. The covariance of X2 with X1 is -0.113 for males and -0.081 for females. Clearly they are not constrained to equality.

Male Female
Estimate S.E. Estimate S.E.

X2 WITH
X1 -0.113 0.057 -0.081 0.064

Looking at the means of the latent variables you probably notice right away that for males, both latent variables have a mean of zero. In contrast the estimated means for females are different from zero (although the difference is not statistically significant in this case). This is related to identification of the model. Instead of estimating the actual mean for each group, the difference between the groups is estimated. To do this

the mean for one of the groups is fixed to some arbitrary value, typically zero.

Male Female

Estimate S.E. Estimate S.E.

Means

X1 0.000 0.000 -0.467 0.167

X2 0.000 0.000 0.486 0.891

For categorical observed variables Mplus gives thresholds of the observed variables rather than intercepts (Threshold = -1 * intercept). Like the intercepts of the observed variables in the continuous example above, the thresholds are constrained to equality across groups.

Male Female

Estimate S.E. Estimate S.E.

Thresholds

A1D$1 1.608 0.073 1.608 0.073

A2D$1 -0.232 0.045 -0.232 0.045

A3D$1 0.261 0.045 0.261 0.045

B1D$1 -1.153 0.056 -1.153 0.056

B2D$1 -0.880 0.050 -0.880 0.050

B3D$1 -1.338 0.062 -1.338 0.062

Below we see that the variances of the latent variables are allowed to be different across groups.

Male Female

Estimate S.E. Estimate S.E.

Variances

X1 0.238 0.137 0.195 0.136

X2 0.748 0.088 1.051 1.139

Similar to the means of the latent variables, the scales are fixed in the first group (although to one in this case), and estimated in the second group. For categorical observed variables, the scale factors relate to the variance of the continuous latent response variable underlying the observed values (which are categorical). The scale factor is fixed to one rather than zero because scale coefficients are multiplicative, rather than additive. Fixing the scales to one in one group, and estimating them the other groups allows the variance of the latent response variable to be different

across groups.

	Male		Female	
	Estimate	S.E.	Estimate	S.E.
Scales				
A1D	1.000	0.000	1.031	0.109
A2D	1.000	0.000	1.102	0.726
A3D	1.000	0.000	1.122	0.401
B1D	1.000	0.000	0.813	0.442
B2D	1.000	0.000	0.919	0.535
B3D	1.000	0.000	0.875	0.392