

# What is the uniform distribution and what are its key characteristics?

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The uniform distribution, also known as the rectangular distribution, is a probability distribution in which all possible outcomes are equally likely to occur. In other words, the probability of obtaining any given value within the range of the distribution is the same. This distribution is often represented by a straight horizontal line, indicating that the likelihood of any outcome is constant. Its key characteristics include a symmetrical shape, where the probability is evenly distributed across the range of values, and a constant probability density function, meaning that the probability remains the same regardless of the value being considered. The uniform distribution is commonly used in situations where there is no prior knowledge or bias towards any specific outcome.

## **An Introduction to the Uniform Distribution**

**The uniform distribution is a probability distribution in which every value between an interval from  $a$  to  $b$  is equally likely to occur.**

**If a  $X$  follows a uniform distribution, then the probability that  $X$  takes on a value between  $x_1$  and  $x_2$  can be found by the following formula:**

$$P(x_1 < X < x_2) = (x_2 - x_1) / (b - a)$$

**where:**

**$x_1$ : the lower value of interest**  
 **$x_2$ : the upper value of interest**  
 **$a$ : the minimum possible value**  
 **$b$ : the maximum possible value**

**For example, suppose the weight of dolphins is uniformly distributed between 100 pounds and 150**

**pounds.**

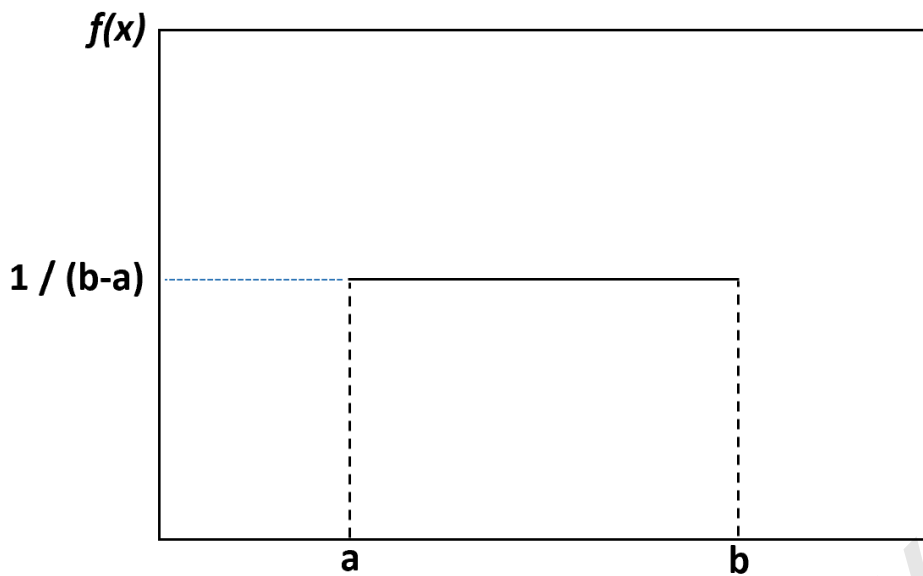
**If we randomly select a dolphin at random, we can use the formula above to determine the probability that the chosen dolphin will weigh between 120 and 130 pounds:**

$$P(120 < X < 130) = (130 - 120) / (150 - 100) \\ P(120 < X < 130) = 10 / 50 \\ P(120 < X < 130) = 0.2$$

**The probability that the chosen dolphin will weigh between 120 and 130 pounds is 0.2.**

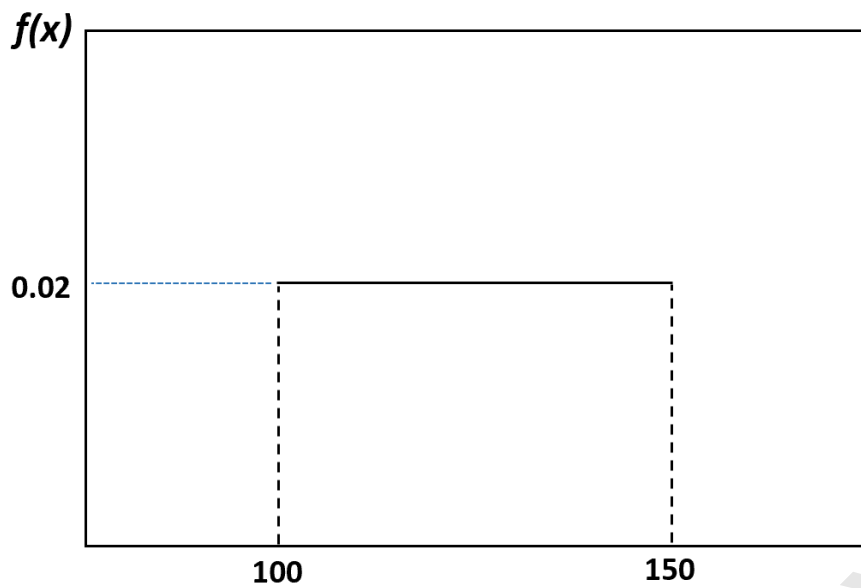
**Visualizing the Uniform Distribution**

**If we create a density plot to visualize the uniform distribution, it would look like the following plot:**

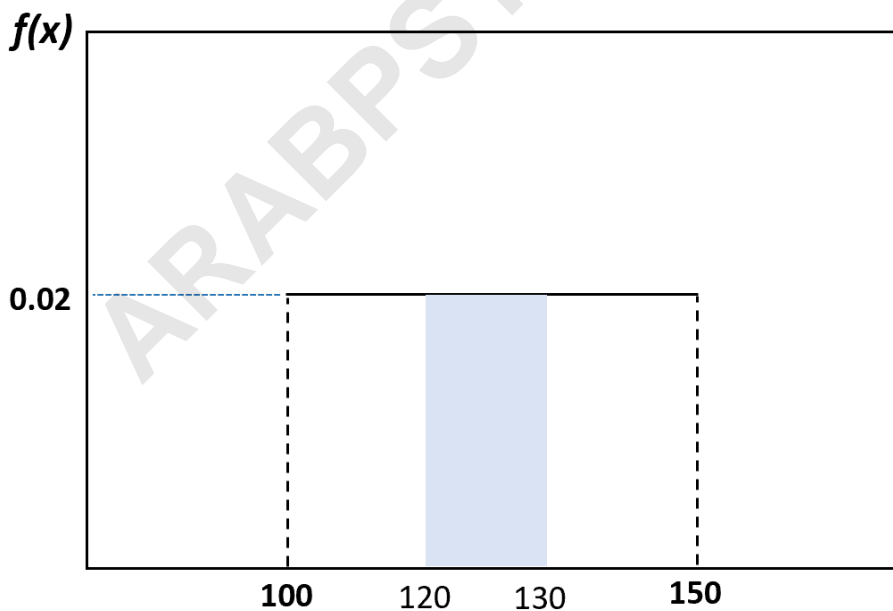


Every value between the lower bound  $a$  and upper bound  $b$  is equally likely to occur and any value outside of those bounds has a probability of zero.

For example, in our previous example we said the weight of dolphins is uniformly distributed between 100 pounds and 150 pounds. Here's how to visualize that distribution:



**And the probability that a randomly selected dolphin weighs between 120 and 130 pounds can be visualized as follows:**



## Properties of the Uniform Distribution

The uniform distribution has the following properties:

Mean:  $(a + b) / 2$  Median:  $(a + b) / 2$  Standard Deviation:  
 $\sqrt{(b - a)^2 / 12}$  Variance:  $(b - a)^2 / 12$

For example, suppose the weight of dolphins is uniformly distributed between 100 pounds and 150 pounds.

We could calculate the following properties for this distribution:

Mean weight:  $(a + b) / 2 = (150 + 100) / 2 = 125$  Median weight:  $(a + b) / 2 = (150 + 100) / 2 = 125$  Standard Deviation of weight:  $\sqrt{(150 - 100)^2 / 12} = 14.43$  Variance of weight:  $(150 - 100)^2 / 12 = 208.33$

## Uniform Distribution Practice Problems

Use the following practice problems to test your knowledge of the uniform distribution.

**Question 1:** A bus shows up at a bus stop every 20 minutes. If you arrive at the bus stop, what is the probability that the bus will show up in 8 minutes or

less?

**Solution 1:** The minimum amount of time you'd have to wait is 0 minutes and the maximum amount is 20 minutes. The lower value of interest is 0 minutes and the upper value of interest is 8 minutes.

Thus, we'd calculate the probability as:

$$P(0 < X < 8) = (8-0) / (20-0) = 8/20 = 0.4.$$

**Question 2:** The length of an NBA game is uniformly distributed between 120 and 170 minutes. What is the probability that a randomly selected NBA game lasts more than 155 minutes?

**Solution 2:** The minimum time is 120 minutes and the maximum time is 170 minutes. The lower value of interest is 155 minutes and the upper value of interest is 170 minutes.

Thus, we'd calculate the probability as:

$$P(155 < X < 170) = (170-155) / (170-120) = 15/50 = 0.3.$$

**Question 3:** The weight of a certain species of frog is

**uniformly distributed between 15 and 25 grams. If you randomly select a frog, what is the probability that the frog weighs between 17 and 19 grams?**

**Solution 3: The minimum weight is 15 grams and the maximum weight is 25 grams. The lower value of interest is 17 grams and the upper value of interest is 19 grams.**

**Thus, we'd calculate the probability as:**

$$P(17 < X < 19) = (19-17) / (25-15) = 2/10 = 0.2.$$

**Note: We can use the to check our answers for each of these problems.**