

What is the triangular distribution and how is it used in statistics?

Authored by
stats writer

April 25, 2024

RECOMMENDED CITATION

stats writer (2024). *What is the triangular distribution and how is it used in statistics?*. PSYCHOLOGICAL SCALES. Retrieved from <https://scales.arabpsychology.com/?p=139024>

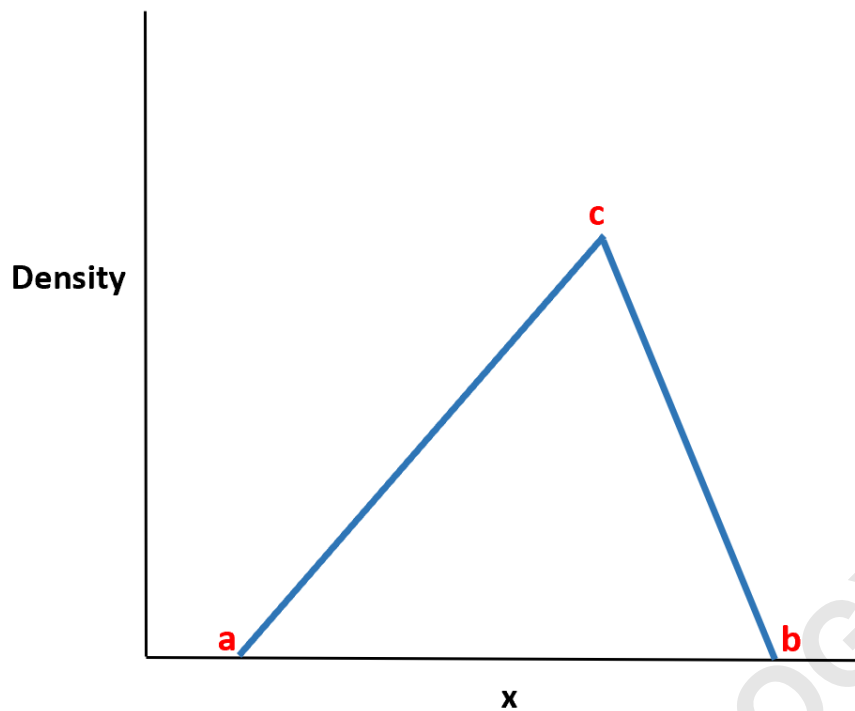
The triangular distribution is a commonly used probability distribution in statistics that is characterized by a triangular-shaped curve. It is often used to model situations where the data is bounded and has a peak at a certain point. This distribution is defined by three parameters: a minimum value, a maximum value, and a mode value. The mode value represents the most likely outcome, while the minimum and maximum values represent the boundaries within which the data is expected to fall. The triangular distribution is useful in statistics as it allows for the estimation of probabilities and the analysis of data with known boundaries. It is commonly used in decision-making, risk analysis, and simulation studies.

An Introduction to the Triangular Distribution

The triangular distribution is a continuous probability distribution with a probability density function shaped like a triangle.

It is defined by three values:

The minimum value a The maximum value b The peak value c

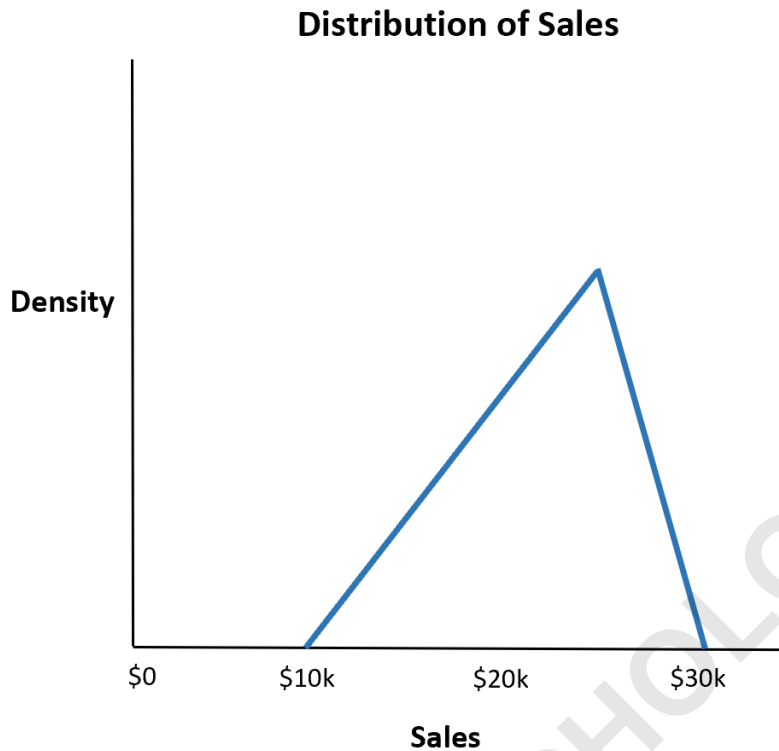


The name of the distribution comes from the fact that the probability density function is shaped like a triangle.

It turns out that this distribution is extremely useful in the real world because we can often estimate the minimum value (a), the maximum value (b), and the most likely value (c) that a will take on, so we can often model the behavior of random variables by using a triangular distribution with the knowledge of just these three values.

For example, a restaurant might estimate that their total sales for the upcoming week will be a minimum of

\$10,000, a maximum of \$30,000, and most likely \$25,000.



Using just these three numbers, they could use a triangular distribution to find the probability that they'll achieve a certain number of sales.

Properties of the Triangular Distribution

The triangular distribution has the following properties:

PDF:

$$p_X(x) = \begin{cases} 0 & \text{for } x < a \\ \frac{2(x-a)}{(b-a)(c-a)} & \text{for } a \leq x < c \\ \frac{2}{b-a} & \text{for } x = c \\ \frac{2(b-x)}{(b-a)(b-c)} & \text{for } c < x \leq b \\ 0 & \text{for } b < x \end{cases}$$

CDF:

$$F(x) = \begin{cases} 0 & \text{for } x \leq a \\ \frac{(x-a)^2}{(b-a)(c-a)} & \text{for } a \leq x \leq c \\ 1 - \frac{(b-x)^2}{(b-a)(b-c)} & \text{for } c < x < b \\ 1 & \text{for } b \leq x \end{cases}$$

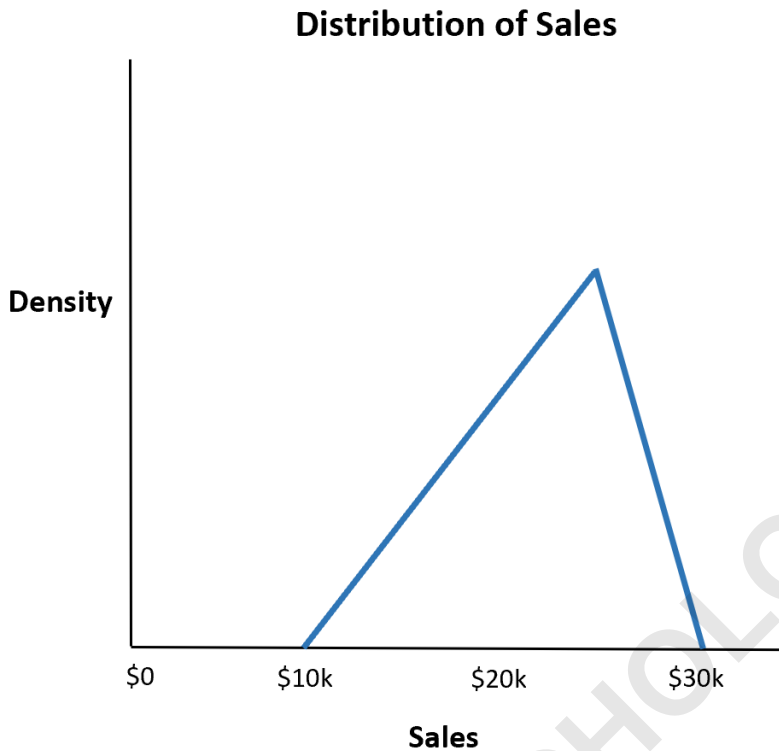
Mean: $(a + b + c) / 3$

Mode: c

Example of Using the Triangular Distribution

Let's return to the example from earlier. Suppose a restaurant estimates that their total sales for the upcoming week will be a minimum of \$10,000, a

maximum of \$30,000, and most likely \$25,000.



What is the probability that the restaurant makes less than \$20,000 total sales?

To answer this, we can let x = total sales. We know that x is between the minimum value a of \$10k and the peak value c of \$25k.

Thus, according to the PDF we can use the following equation to find the probability that the restaurant makes less than \$20,000 total sales:

$$P(X < \$20,000) = (x-a)^2 / ((b-a)(c-a))P(X < \$20,000) = (20,000-10,000)^2 / ((30,000-10,000)(25,000-10,000))P(X < \$20,000) = .333$$

The probability that the restaurant makes less than \$20,000 total sales is .333.

What is the mean expected sales for the restaurant?

We can calculate the mean expected sales using the formula for the mean given earlier:

$$\text{Mean} = (a + b + c) / 3 \text{Mean} = (\$10,000 + \$30,000 + \$25,000) / 3 \text{Mean} = \$21,667$$

The mean expected sales is 21,667.

The following tutorials provide introductions to other commonly used distributions: