

“What is the statistical significance and relationship between the standard normal distribution and statistical analysis?”

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The standard normal distribution is a type of probability distribution that is used in statistical analysis to represent a wide range of data. It is a bell-shaped curve that is symmetric, with a mean of 0 and a standard deviation of 1. This distribution is significant in statistical analysis as it allows for the calculation of probabilities and the assessment of the likelihood of certain outcomes. It also serves as a reference point for other distributions, making comparisons and statistical tests more meaningful. The relationship between the standard normal distribution and statistical analysis lies in the fact that it is commonly used as a basis for various statistical tests and techniques, such as hypothesis testing, confidence intervals, and regression analysis. By understanding the characteristics and properties of the standard normal distribution, researchers can effectively apply statistical methods to analyze and interpret data in a meaningful way.

Statistics - Standard Normal Distribution

The standard normal distribution is a normal distribution where the mean is 0 and the standard deviation is 1.

Standard Normal Distribution

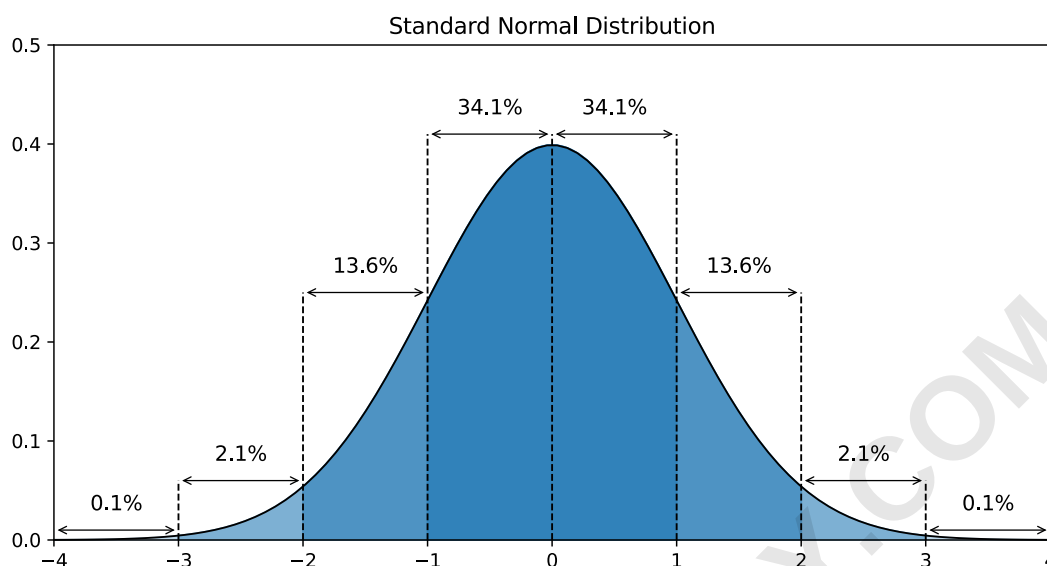
Normally distributed data can be transformed into a standard normal distribution.

Standardizing normally distributed data makes it easier to compare different sets of data.

The standard normal distribution is used for:

Calculating confidence intervalsHypothesis tests

Here is a graph of the standard normal distribution with probability values (p-values) between the standard deviations:



Standardizing makes it easier to calculate probabilities.

The functions for calculating probabilities are complex and difficult to calculate by hand.

Typically, probabilities are found by looking up tables of pre-calculated values, or by using software and programming.

The standard normal distribution is also called the 'Z-distribution' and the values are called 'Z-values' (or Z-scores).

Z-Values

Z-values express how many standard deviations from the mean a value is.

The formula for calculating a Z-value is:

$$Z = \frac{x - \mu}{\sigma}$$

(x) is the value we are standardizing, (μ) is the mean, and (σ) is the standard deviation.

For example, if we know that:

The mean height of people in Germany is 170 cm (μ)

The standard deviation of the height of people in Germany is 10 cm (σ)

Bob is 200 cm tall ((x))

Bob is 30 cm taller than the average person in Germany.

30 cm is 3 times 10 cm. So Bob's height is 3 standard deviations larger than mean height in Germany.

Using the formula:

$$(displaystyle Z = \frac{x-\mu}{\sigma} = \frac{200-170}{10} = \frac{30}{10} = \underline{3})$$

The Z-value of Bob's height (200 cm) is 3.

Finding the P-value of a Z-Value

Using a Z-table or programming we can calculate how many people Germany are shorter than Bob and how many are taller.

Example

With Python use the Scipy Stats library `norm.cdf()` function find the probability of getting less than a Z-value of 3:

```
import scipy.stats as stats
```

```
print(stats.norm.cdf(3))
```

Example

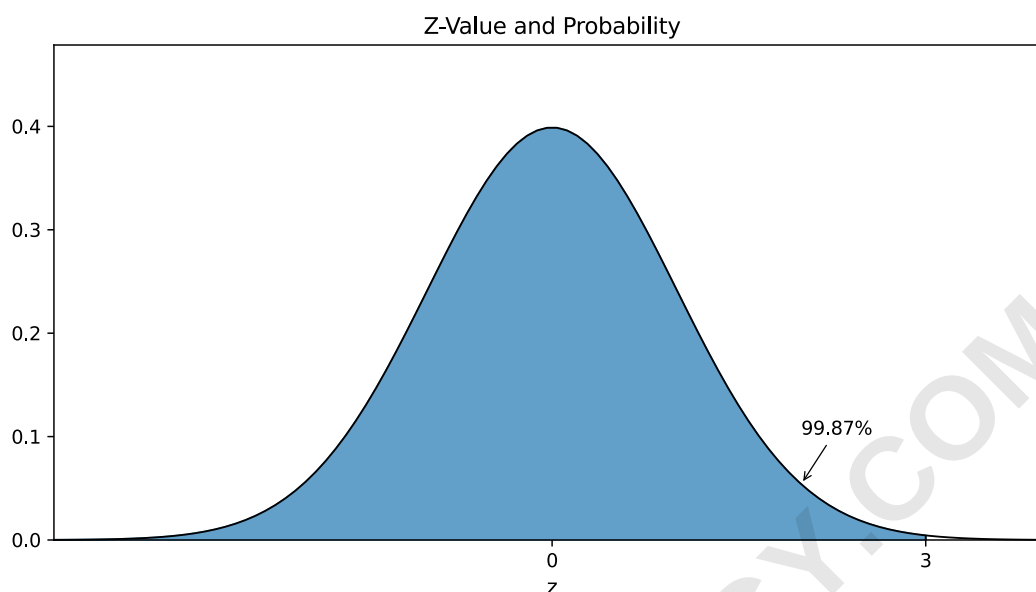
With R use the built-in `pnorm()` function find the probability of getting less than a Z-value of 3:

```
pnorm(3)
```

Using either method we can find that the probability is (approx 0.9987), or (99.87%)

Which means that Bob is taller than 99.87% of the people in Germany.

Here is a graph of the standard normal distribution and a Z-value of 3 to visualize the probability:



These methods find the p-value up to the particular z-value we have.

To find the p-value above the z-value we can calculate 1 minus the probability.

So in Bob's example, we can calculate $1 - 0.9987 = 0.0013$, or 0.13%.

Which means that only 0.13% of Germans are taller than Bob.

Finding the P-Value Between Z-Values

If we instead want to know how many people are between 155 cm and 165 cm in Germany using the same example:

The mean height of people in Germany is 170 cm (μ)

The standard deviation of the height of people in Germany is 10 cm (σ)

Now we need to calculate Z-values for both 155 cm and 165 cm:

$$Z = \frac{x - \mu}{\sigma} = \frac{155 - 170}{10} = \frac{-15}{10} = \underline{-1.5}$$

The Z-value of 155 cm is -1.5

$$Z = \frac{x - \mu}{\sigma} = \frac{165 - 170}{10} = \frac{-5}{10} = \underline{-0.5}$$

The Z-value of 165 cm is -0.5

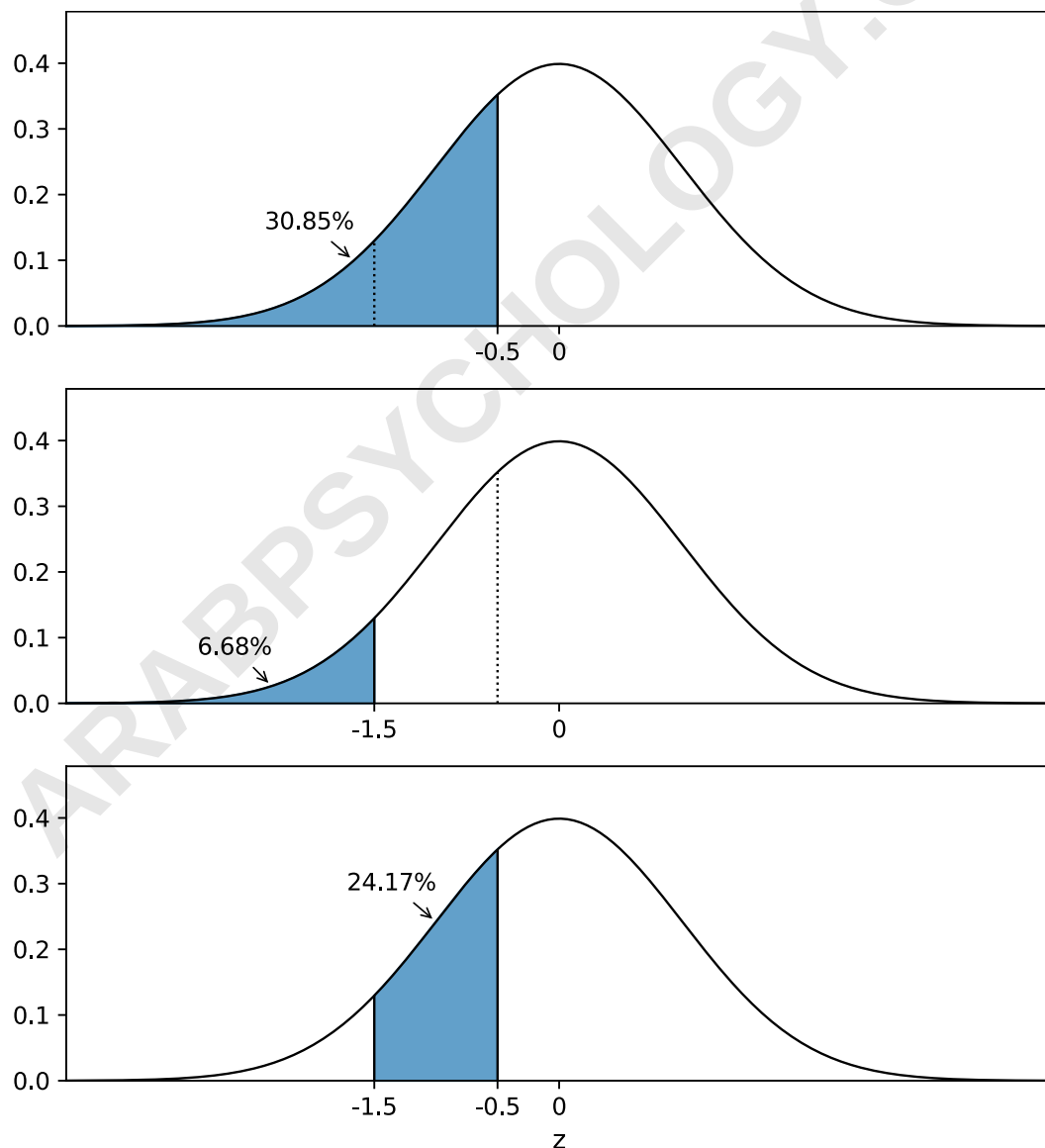
Using the Z-table or programming we can find that the p-value for the two z-values:

The probability of a z-value smaller than -0.5 (shorter than 165 cm) is 30.85% The probability of a z-value smaller than -1.5 (shorter than 155 cm) is 6.68%

Subtract 6.68% from 30.85% to find the probability of getting a z-value between them.

$$30.85\% - 6.68\% = 24.17\%$$

Here is a set of graphs illustrating the process:



Finding the Z-value of a P-Value

You can also use p-values (probability) to find z-values.

For example:

"How tall are you if you are taller than 90% of Germans?"

The p-value is 0.9, or 90%.

Using a Z-table or programming we can calculate the z-value:

Example

With Python use the Scipy Stats library `norm.ppf()` function find the z-value separating the top 10% from the bottom 90%:

```
import scipy.stats as stats
```

```
print(stats.norm.ppf(0.9))
```

Example

With R use the built-in `qnorm()` function find the z-value separating the top 10% from the bottom 90%:

```
qnorm(0.9)
```

Using either method we can find that the Z-value is (approx 1.281)

Meaning that a person that is 1.281 standard deviations taller than the mean height of Germans is taller than 90% of Germans.

We then use the formula to calculate the height ((x)) based on a mean ((mu)) of 170 cm and standard deviation ((sigma)) of 10 cm:

$$Z = \frac{x - \mu}{\sigma}$$

$$1.281 = \frac{x - 170}{10}$$

$$(1.281 \cdot 10 = x - 170)$$

$$(12.81 = x - 170)$$

$$(12.81 + 170 = x)$$

$$(\underline{182.81} = x)$$

So we can conclude that:

"You have to be at **least** 182.81 cm tall to be taller than 90% of Germans"

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