

# What is the shape of a binomial distribution and how does it relate to its parameters?

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## RECOMMENDED CITATION

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A binomial distribution is a type of probability distribution that represents the number of successes in a series of independent trials with a fixed probability of success. It is characterized by two parameters, the number of trials ( $n$ ) and the probability of success ( $p$ ). The shape of a binomial distribution is typically bell-shaped, with the highest point at the most likely number of successes. The distribution is symmetrical when  $p=0.5$  and becomes increasingly skewed as  $p$  moves away from 0.5. The parameters of a binomial distribution directly affect its shape, with larger values of  $n$  resulting in a narrower and taller distribution, and larger values of  $p$  resulting in a higher peak. In summary, the shape of a binomial distribution is determined by its parameters, and it provides a visual representation of the probability of different outcomes in a series of independent trials.

## Understanding the Shape of a Binomial Distribution

The binomial distribution describes the probability of obtaining  $k$  successes in  $n$  binomial experiments.

If a random variable  $X$  follows a binomial distribution, then the probability that  $X = k$  successes can be found by the following formula:

$$P(X=k) = nCk * p^k * (1-p)^{n-k}$$

where:

$n$ : number of trials  
 $k$ : number of successes  
 $p$ : probability of success on a given trial  
 $nCk$ : the number of ways to obtain  $k$  successes in  $n$  trials

The binomial probability distribution tends to be bell-shaped when one or more of the following two

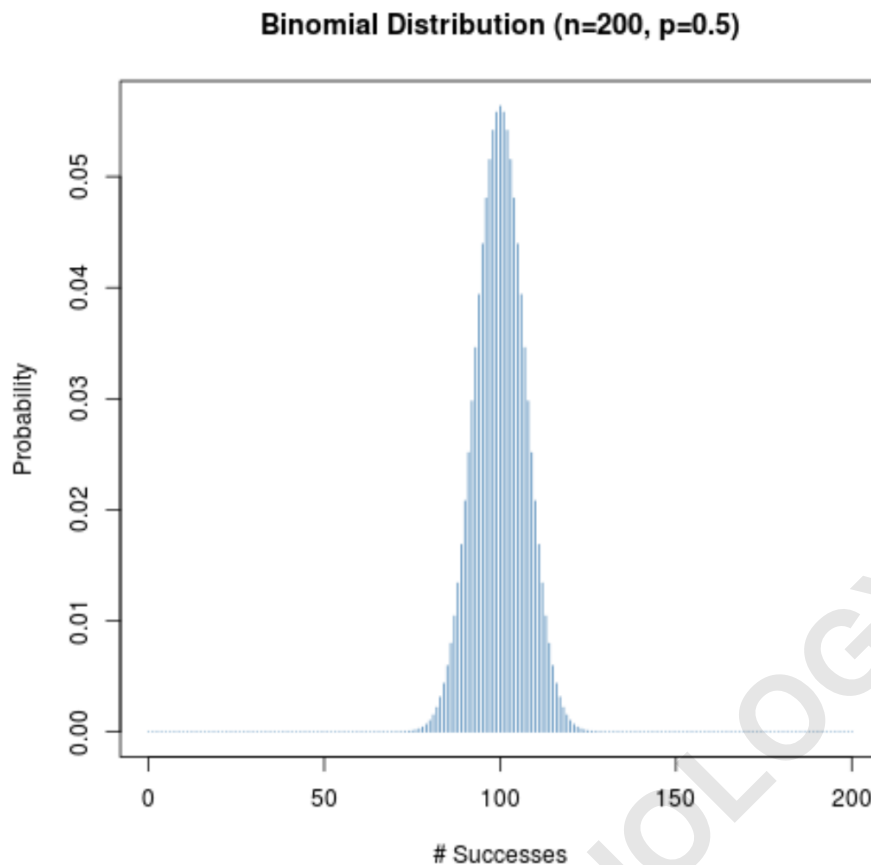
## **conditions occur:**

- 1. The sample size (n) is large.**
- 2. The probability of success on a given trial (p) is close to 0.5.**

**However, the binomial probability distribution tends to be skewed when neither of these conditions occur. To illustrate this, consider the following examples:**

**Example 1: Sample Size (n) is Large**

**The following chart displays the probability distribution for when  $n = 200$  and  $p = 0.5$ .**

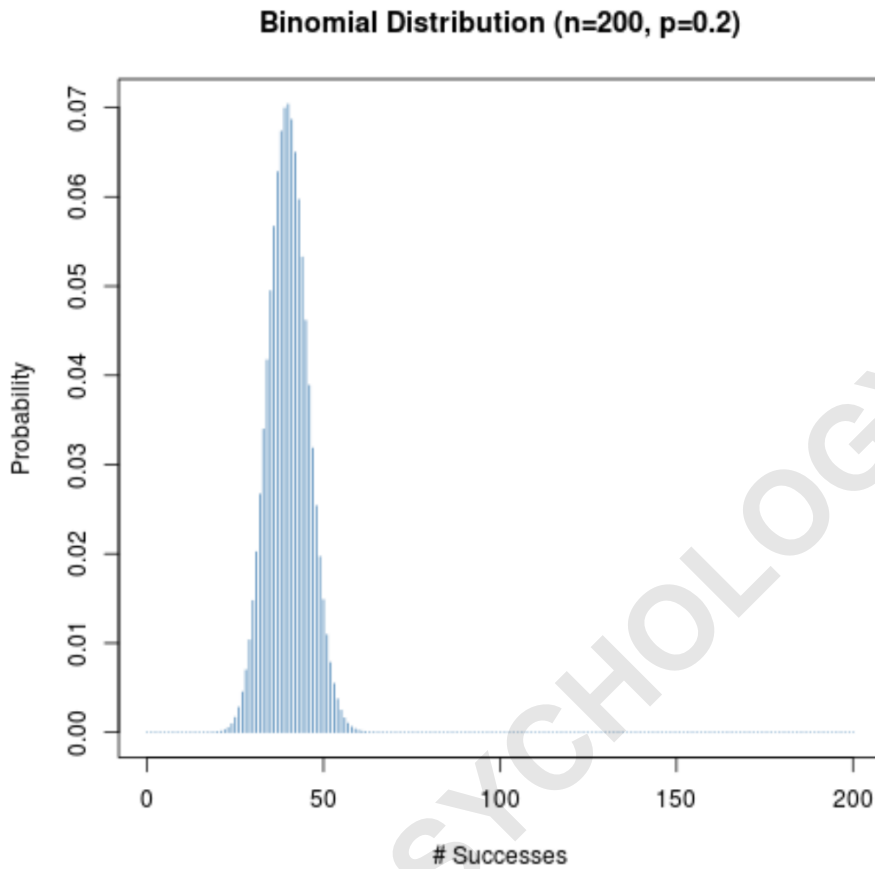


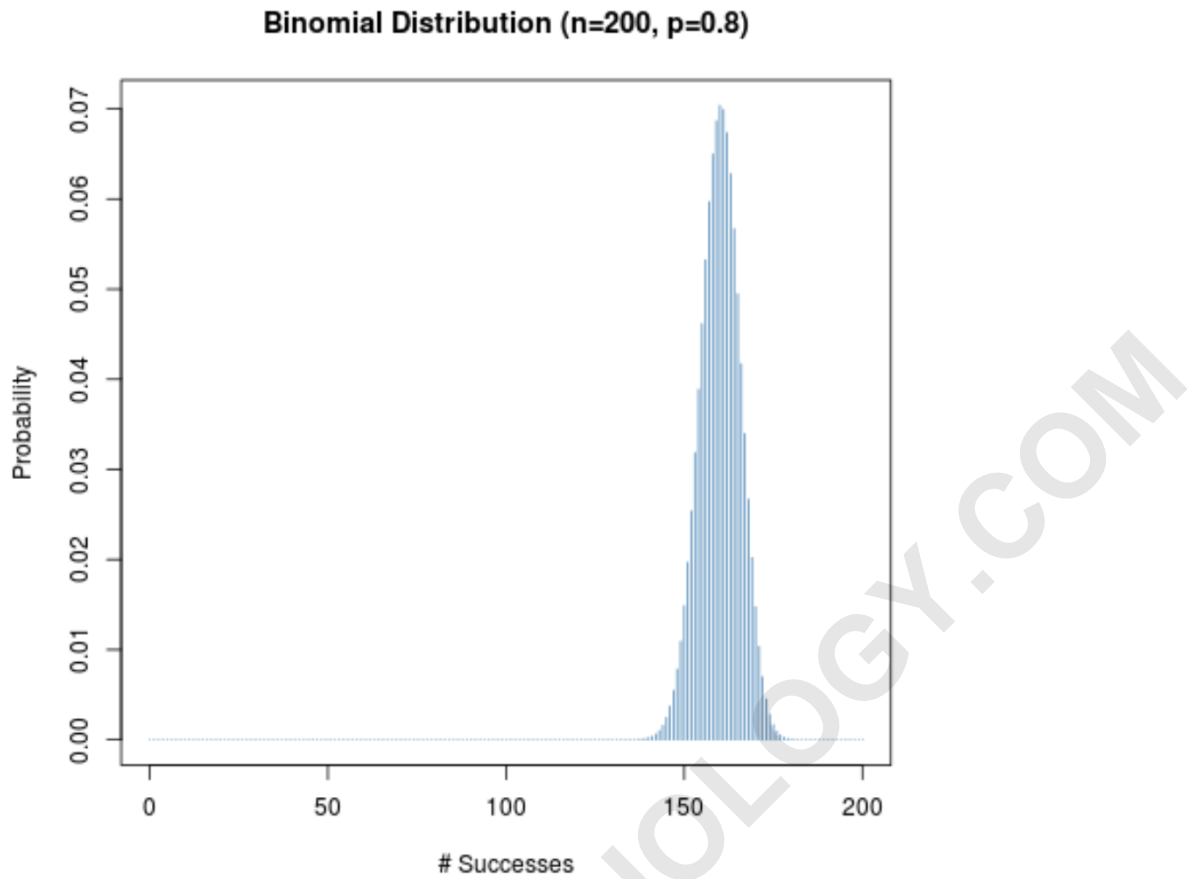
**The x-axis displays the number of successes during 200 trials and the y-axis displays the probability of that number of successes occurring.**

**Since both (1) the sample size is large and (2) the probability of success on a given trial is close to 0.5, the probability distribution is bell-shaped.**

**Even when the probability of success on a given trial ( $p$ ) is not close to 0.5, the probability distribution will still be bell-shaped as long as the sample size ( $n$ ) is large.**

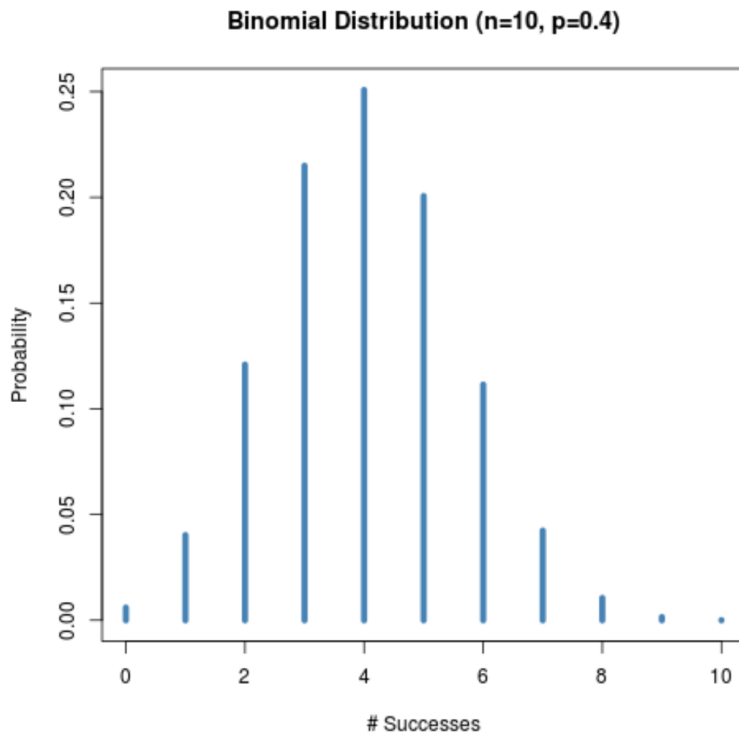
To illustrate this, consider the following two scenarios when  $p = 0.2$  and  $p = 0.8$ .





**Example 2: Probability of Success (p) is Close to 0.5**

**The following chart displays the probability distribution for when  $n = 10$  and  $p = 0.4$ .**

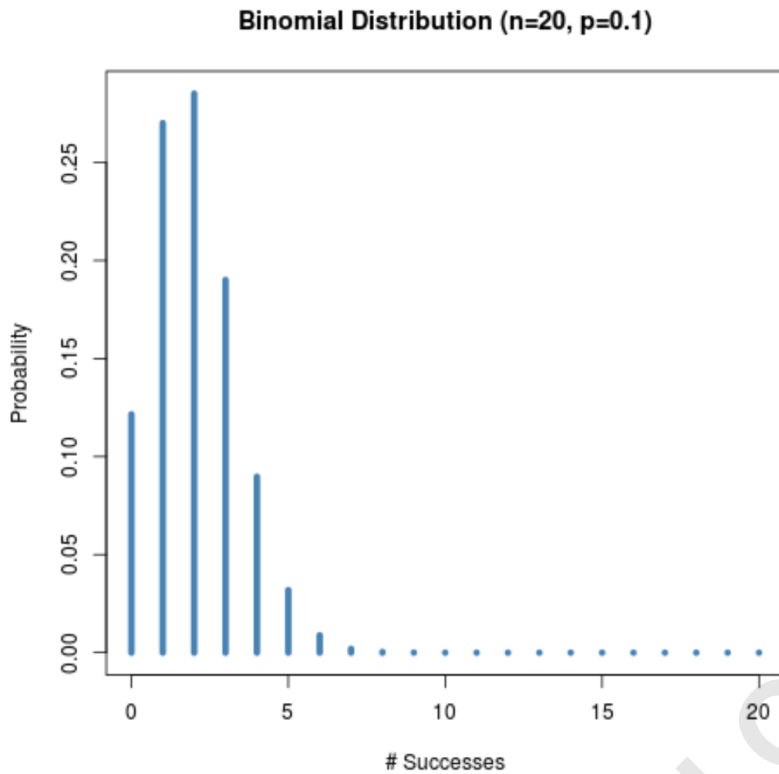


Although the sample size ( $n = 10$ ) is small, the probability distribution is still bell-shaped because the probability of success on a given trial ( $p = 0.4$ ) is close to 0.5

### Example 3: Skewed Binomial Distributions

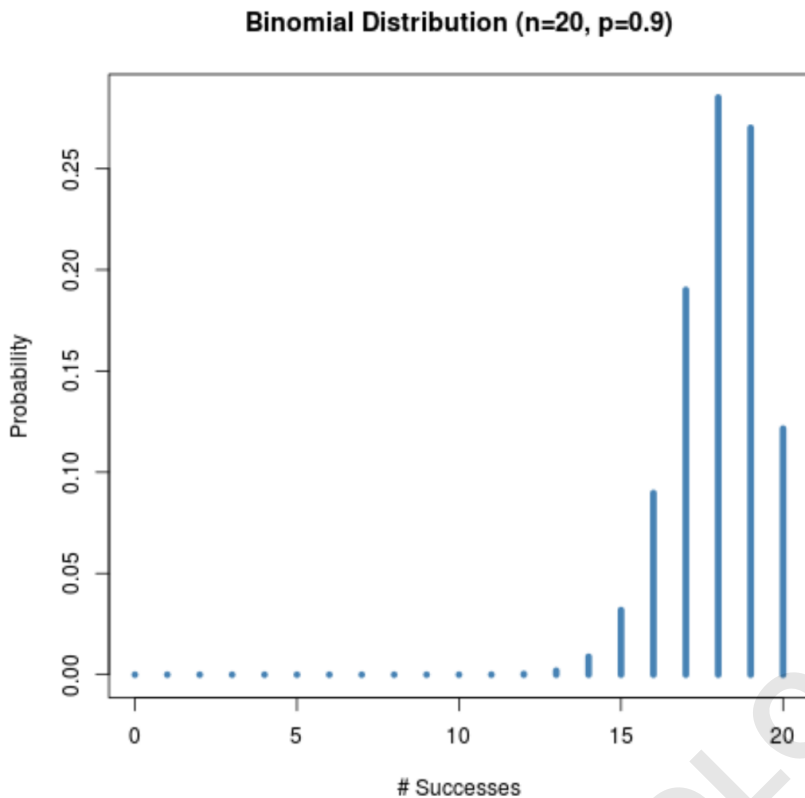
When neither (1) the sample size is large nor (2) the probability of success on a given trial is close to 0.5, the binomial probability distribution will be skewed to the left or right.

For example, the following plot shows the probability distribution when  $n = 20$  and  $p = 0.1$ .



**Notice how the distribution is skewed to the right.**

**And the following plot shows the probability distribution when  $n = 20$  and  $p = 0.9$ .**



**Notice how the distribution is skewed to the left.**

### Ending Notes

**Each of the charts in this post were created using the statistical programming language R. Learn how to plot your own binomial probability distributions in R using [this tutorial](#).**