

How to Reduce Margin of Error by Increasing Sample Size

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The relationship between sample size and margin of error in statistical analysis is fundamentally **inverse**. This means that as researchers increase the number of observations included in a study--the sample size--the corresponding margin of error will decrease. This crucial dynamic is rooted in the principle of representativeness: a larger sample is generally considered more reflective of the overall population from which it was drawn.

Consequently, larger samples yield estimates that are closer to the true population parameters, providing more reliable results and narrower ranges of uncertainty. Conversely, relying on a small sample introduces greater uncertainty. When the sample size is limited, the results are less reliable, leading to a significant increase in the potential margin of error. Understanding this trade-off is essential for designing effective statistical studies, as it dictates the balance between the cost of data collection and the desired precision of the outcome.

The Role of Sample Statistics in Population Inference

In the field of statistics, our primary goal is often to estimate unknown characteristics of a large group, known as the population. We are typically interested in estimating core parameters, such as the true population proportion (P) or the true population mean (μ). Since surveying an entire population is often impractical or impossible, we must rely on subsets of data.

To achieve these estimates, researchers must collect a representative sample of data. From this sample, we calculate sample statistics--the sample proportion (p) or the sample mean (\bar{x}). These sample statistics serve as point estimates for the corresponding population parameters.

However, a point estimate alone does not convey the potential for sampling error. Therefore, we construct a confidence interval around the estimate to quantify the uncertainty and precision involved in the estimation process. The width of this interval is directly influenced by the calculated margin of error.

Mathematical Foundation: Confidence Interval Formulas

The mathematical relationship between precision and sample size becomes clear when examining the formulas used to calculate confidence intervals. The structure of these formulas explicitly demonstrates how sample size acts as a divisor, thereby controlling the overall magnitude of the error term.

We utilize the following standard formula to calculate a confidence interval for a population proportion:

$$\text{Confidence Interval} = p \pm z^* \sqrt{p(1-p) / n}$$

where the variables are defined as follows:

p: The calculated **sample proportion**.

z: The **z-value** (critical value) associated with the chosen confidence level.

n: The **sample size**, the total number of observations collected.

Similarly, to calculate a confidence interval for a population mean, we employ a closely related structure:

Confidence Interval = $\bar{x} \pm z^*(s/\sqrt{n})$

where these variables represent:

\bar{x} ; The calculated **sample mean**.

z: The **z-value** (critical value) corresponding to the desired confidence level.

s: The **sample standard deviation**.

n: The **sample size**.

In both mathematical frameworks, the inverse relationship between the sample size (n) and the resulting margin of error is mathematically undeniable.

The primary takeaway is clear: the larger the value of 'n' (the sample size), the smaller the divisor applied to the variance term, resulting in a reduced margin of error. Conversely, a reduction in the sample size leads directly to an expansion of the error boundary.

To solidify this understanding, let us examine two practical examples demonstrating the significant impact of changing sample size on the margin of error for both population proportions and means.

Example 1: The Impact of Sample Size on Population Proportion Estimates

We begin by analyzing the estimation of a population proportion, a common task in survey research. Recall the formula, where the term following the plus/minus sign represents the margin of error:

Confidence Interval = $p \pm z^*\sqrt{p(1-p)} / n$

Specifically, the margin of error is identified as:

Confidence Interval = $p \pm z^*\sqrt{p(1-p)} / n$

Note the placement of the variable 'n' (the sample size) in the denominator of the square root term. Because 'n' serves as a divisor, increasing its value necessarily reduces the value of the entire

fraction. A smaller result for the margin of error leads directly to a narrower, more precise confidence interval.

Let us consider an initial scenario where a simple random sample yields the following data points, using a 95% confidence level ($z = 1.96$):

p: 0.6

n: 25 (The initial, small sample size)

Here is the calculation for the 95% confidence interval for the population proportion:

$$\text{Confidence Interval} = p \pm z^* \sqrt{p(1-p)} / n$$

$$\text{Confidence Interval} = .6 \pm 1.96^* \sqrt{.6(1-.6)} / 25$$

$$\text{Confidence Interval} = .6 \pm 0.192$$

$$\text{Confidence Interval} =$$

Increasing the Sample Size for Proportion Estimation

Now observe the profound effect if the research team were able to increase the sample size significantly, while keeping the sample proportion and confidence level constant. We will raise 'n' from 25 to 200.

p: 0.6

n: 200 (The substantially increased sample size)

Here is the calculation for the new 95% confidence interval for the population proportion:

$$\text{Confidence Interval} = p \pm z^* \sqrt{p(1-p)} / n$$

$$\text{Confidence Interval} = .6 \pm 1.96^* \sqrt{.6(1-.6)} / 200$$

$$\text{Confidence Interval} = .6 \pm 0.068$$

$$\text{Confidence Interval} =$$

By increasing the sample size eightfold (from 25 to 200), the margin of error was drastically reduced from 0.192 to 0.068. This transition results in a much more precise and narrow confidence interval, significantly improving the quality of the estimate.

Example 2: The Relationship in Estimating a Population Mean

We now turn our attention to the confidence interval calculation for the population mean. This formula demonstrates the same fundamental inverse relationship between precision and the volume of data collected.

The formula used is:

$$\text{Confidence Interval} = \bar{x} \pm z^*(s/\sqrt{n})$$

The portion in red represents the **margin of error**:

$$\text{Confidence Interval} = \bar{x} \pm z^*(s/\sqrt{n})$$

As in the previous example, the sample size (n) is found in the denominator, specifically under the square root in the standard error term (s/\sqrt{n}). When ' n ' increases, the standard error decreases significantly, thereby shrinking the overall margin of error and yielding a narrower confidence interval.

Consider an initial scenario derived from a simple random sample, using a 95% confidence level:

\bar{x} : 15 (The sample mean)

s : 4 (The sample standard deviation)

n : 25 (The initial sample size)

Here is the calculation for the 95% confidence interval for the population mean:

$$\text{Confidence Interval} = \bar{x} \pm z^*(s/\sqrt{n})$$

$$\text{Confidence Interval} = 15 \pm 1.96*(4/\sqrt{25})$$

$$\text{Confidence Interval} = 15 \pm 1.568$$

$$\text{Confidence Interval} =$$

The Effect of Expanded Sample Size on Mean Estimation

If we were to increase the sample size while maintaining the other parameters, we expect a corresponding reduction in uncertainty. Let us boost ' n ' from 25 to 200.

\bar{x} : 15

s : 4

n : 200 (The new sample size)

The resulting 95% confidence interval calculation is as follows:

$$\text{Confidence Interval} = \bar{x} \pm z^*(s/\sqrt{n})$$

$$\text{Confidence Interval} = 15 \pm 1.96*(4/\sqrt{200})$$

$$\text{Confidence Interval} = 15 \pm 0.554$$

$$\text{Confidence Interval} =$$

This scenario clearly illustrates the benefit of increasing the amount of data collected. By

increasing the sample size to 200, the margin of error dropped significantly from 1.568 to 0.554, yielding a substantially more focused and precise confidence interval.

Conclusion: Optimizing Sample Size for Precision

The foundational principle demonstrated across both examples is the efficiency gained by increasing the sample size: larger samples lead to more stable statistical estimates and dramatically reduce the inherent uncertainty captured by the margin of error. Researchers must strategically select a sample size that balances the practical limitations of data collection (cost and time) against the desired level of precision.

It is important to remember that the relationship is non-linear, governed by the square root of n . This implies that there are diminishing returns as the sample size grows very large; reducing the margin of error by half, for instance, requires quadrupling the sample size.

Further Exploration of Confidence Intervals

The following tutorials provide additional information and practical methods for calculating and interpreting confidence intervals for a proportion:

The following tutorials provide additional information and in-depth guides concerning confidence intervals for a mean: