What is the meaning of Z-scores and can you provide examples of their interpretation?

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Z-scores, also known as standard scores, are a statistical measurement that represents the number of standard deviations a data point is above or below the mean of a data set. They are used to compare individual data points to the overall distribution of a data set and to identify outliers. A positive Z-score indicates that a data point is above the mean, while a negative Z-score indicates it is below the mean. For example, if a student's Z-score on a test is +1.5, it means that their score is 1.5 standard deviations above the mean of all students' scores. Z-scores are particularly useful in comparing data sets with different means and standard deviations, as they allow for a standardized comparison. By understanding how far a data point deviates from the mean in terms of standard deviations, we can better interpret and analyze data.

Interpret Z-Scores (With Examples)

In statistics, a z-score tells us how many standard deviations away a given value lies from the mean. We use the following formula to calculate a z-score:

$$z = (X - \mu) / \sigma$$

where:

X is a single raw data valueμ is the meanσ is the standard deviation

A z-score for an individual value can be interpreted as follows:

Positive z-score: The individual value is greater than the mean.Negative z-score: The individual value is less than the mean.A z-score of 0: The individual value is equal to

the mean.

The larger the absolute value of the z-score, the further away an individual value lies from the mean.

The following example shows how to calculate and interpret z-scores.

Example: Calculate and Interpret Z-Scores

Suppose the scores for a certain exam are normally distributed with a mean of 80 and a standard deviation of 4.

Question 1: Find the z-score for an exam score of 87.

We can use the following steps to calculate the z-score:

The mean is $\mu = 80$The standard deviation is $\sigma = 4$The individual value we're interested in is X = 87Thus, $z = (X - \mu) / \sigma = (87 - 80) / 4 = 1.75$.

This tells us that an exam score of 87 lies 1.75 standard deviations *above* the mean.

Question 2: Find the z-score for an exam score of 75.

We can use the following steps to calculate the z-score:

The mean is μ = 80The standard deviation is σ = 4The individual value we're interested in is X = 75Thus, z = (X - μ) / σ = (75 - 80) /4 = -1.25.

Question 3: Find the z-score for an exam score of 80.

We can use the following steps to calculate the z-score:

The mean is μ = 80The standard deviation is σ = 4The individual value we're interested in is X = 80Thus, z = (X - μ) / σ = (80 - 80) /4 = 0.

This tells us that an exam score of 80 is exactly equal to the mean.

Why Are Z-Scores Useful?

Z-scores are useful because they give us an idea of how an individual value compares to the rest of a distribution.

For example, is an exam score of 87 good? Well, that depends on the mean and standard deviation of all exam scores.

If the exam scores for the whole population are normally distributed with a mean of 90 and a standard deviation of 4, we would calculate the z-score for 87 to be:

$z = (X - \mu) / \sigma = (87 - 90) / 4 = -0.75$.

Since this value is negative, it tells us that an exam score of 87 is actually *below* the average exam score for the population. Specifically, an exam score of 87 is 0.75 standard deviations below the mean.

In a nutshell, z-scores give us an idea of how individual values compare to the mean.

How to Calculate Z-Scores in Practice

The following tutorials show step-by-step examples of how to calculate z-scores in different statistical software: