

What is the Law of total probability?

Authored by
stats writer

December 9, 2025

RECOMMENDED CITATION

stats writer (2025). *What is the Law of total probability?*. PSYCHOLOGICAL SCALES.
Retrieved from <https://scales.arabpsychology.com/?p=106786>

The Law of Total Probability is a fundamental theorem of probability theory which states that for any event, the probability of that event is the sum of the probabilities of each of its possible outcomes. This theorem can be used to calculate the probability of an event by summing the probabilities of all its possible outcomes.

In probability theory, **the law of total probability** is a useful way to find the probability of some event A when we don't directly know the probability of A but we do know that events $B_1, B_2, B_3, \dots$ form a partition of the S .

This law states the following:

The Law of Total Probability

If $B_1, B_2, B_3, \dots$ form a partition of the sample space S , then we can calculate the probability of event A as:

$$P(A) = \sum P(A|B_i) \cdot P(B_i)$$

The easiest way to understand this law is with a simple example.

Suppose there are two bags in a box, which contain the following marbles:

Bag 1: 7 red marbles and 3 green marbles

Bag 2: 2 red marbles and 8 green marbles

If we randomly select one of the bags and then randomly select one marble from that bag, what is the probability that it's a green marble?

In this example, let $P(G)$ = probability of choosing a green marble. This is the probability that we're interested in, but we can't compute it directly.

Instead we need to use the conditional probability of G , given some events B where the B_i 's form a partition of the sample space S . In this example, we have the following conditional probabilities:

$$P(G|B_1) = 3/10 = 0.3$$

$$P(G|B_2) = 8/10 = 0.8$$

Thus, using the law of total probability we can calculate the probability of choosing a green marble as:

$$P(G) = \sum P(G|B_i) \cdot P(B_i)$$

$$P(G) = P(G|B_1) \cdot P(B_1) + P(G|B_2) \cdot P(B_2)$$

$$P(G) = (0.3)*(0.5) + (0.8)*(0.5)$$

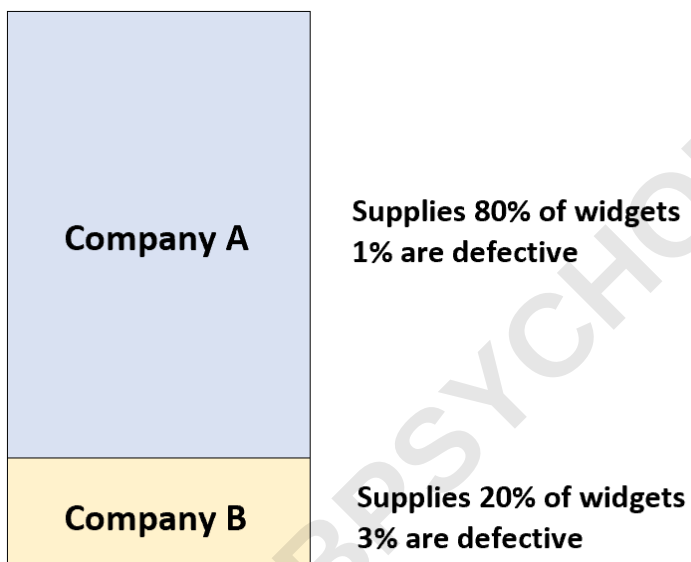
$$P(G) = \mathbf{0.55}$$

If we randomly select one of the bags and then randomly select one marble from that bag, the probability we choose a green marble is **0.55**.

Read through the next two examples to solidify your understanding of the law of total probability.

Example 1: Widgets

Company A supplies 80% of widgets for a car shop and only 1% of their widgets turn out to be defective. Company B supplies the remaining 20% of widgets for the car shop and 3% of their widgets turn out to be defective.



If we let $P(D)$ = the probability of a widget being defective and $P(B_i)$ be the probability that the widget came from one of the companies, then we can compute the probability of buying a defective widget as:

$$P(D) = \sum P(D|B_i)*P(B_i)$$

$$P(D) = P(D|B_1)*P(B_1) + P(D|B_2)*P(B_2)$$

$$P(D) = (0.01)*(0.80) + (0.03)*(0.20)$$

$$P(D) = \mathbf{0.014}$$

If we randomly buy a widget from this car shop, the probability that it will be defective is **0.014**.

Example 2: Forests

Forest A occupies 50% of the total land in a certain park and 20% of the plants in this forest are poisonous. Forest B occupies 30% of the total land and 40% of the plants in it are poisonous. Forest C occupies the remaining 20% of the land and 70% of the plants in it are poisonous.

Forest A	Occupies 50% of total land 20% of plants are poisonous
Forest B	Occupies 30% of total land 40% of plants are poisonous
Forest C	Occupies 20% of total land 70% of plants are poisonous

If we randomly enter this park and pick a plant from the ground, what is the probability that it will be poisonous?

If we let $P(P)$ = the probability of the plant being poisonous, and $P(B_i)$ be the probability that we've entered one of the three forests, then we can compute the probability of a randomly chosen plant being poisonous as:

$$P(P) = \sum P(P|B_i) \cdot P(B_i)$$

$$P(P) = P(P|B_1) \cdot P(B_1) + P(P|B_2) \cdot P(B_2) + P(P|B_3) \cdot P(B_3)$$

$$P(P) = (0.20) \cdot (0.50) + (0.40) \cdot (0.30) + (0.70) \cdot (0.20)$$

$$P(P) = \mathbf{0.36}$$

If we randomly pick a plant from the ground, the probability that it will be poisonous is **0.36**.

The following tutorials provide additional information about topics related to probability: