“What is the interpretation of odds ratios in logistic regression?”

Authored by

stats writer

June 30, 2024

The interpretation of odds ratios in logistic regression refers to the measure of the relationship between a binary outcome variable and one or more independent variables. It is a way to quantify the impact of the independent variables on the odds of the outcome occurring. An odds ratio of 1 indicates no relationship between the variables, a value greater than 1 indicates a positive relationship, and a value less than 1 indicates a negative relationship. The odds ratio can also provide information on the direction and strength of the relationship between the variables. It is commonly used in logistic regression to assess the significance of the independent variables and make predictions about the likelihood of the outcome occurring.

How do I interpret odds ratios in logistic regression? | SPSS FAQ

Introduction

Let's begin with probability. Let's say that the probability of success is .8, thus

p = .8

Then the probability of failure is

q = 1 - p = .2

The odds of success are defined as

odds(success) = p/q = .8/.2 = 4,

that is, the odds of success are 4 to 1. The odds of failure would be

odds(failure) = q/p = .2/.8 = .25.

This looks a little strange but it is really saying that the odds of failure are 1 to 4.

The odds of success and the odds of failure are just reciprocals of one another, i.e.,
1/4 = .25 and 1/.25 = 4.

Next, we will add another variable to the equation so that we can compute and odds ratio.

Another example

This example is adapted from Pedhazur (1997). Suppose
that seven out of 10 males are admitted to an engineering school while three of 10 females
are admitted. The probabilities for admitting a male are,

p = 7/10 = .7 q = 1 - .7 = .3

Here are the same probabilities for females,

p = 3/10 = .3 q = 1 - .3 = .7

Now we can use the probabilities to compute the

admission odds for both males and
females,

odds(male) = .7/.3 = 2.33333
odds(female) = .3/.7 = .42857

Next, we compute the odds ratio for admission,

OR = 2.3333/.42857 = 5.44

Thus, the odds of a male being admitted are 5.44 times greater than for a female.

Logistic regression in SPSS

Here are the SPSS logistic regression commands and output for the example above. In this example admit is coded 1 for yes and 0 for
no,
and gender is coded 1 for male and 0 for female. We have also
included a variable called freq which give the frequency with which each
case occurs. We use the weight by command to weight our cases.
Also, in the interest of saving space, we have included

only the last of the
tables that are presented in the SPSS output. The odds ratio is given in
the right-most column labeled "Exp(B)". The relationship between the odds
ratio and the coefficient (given in the column labeled "B") is explained in the
next section ("About logits").

```
data list list
/admit gender freq.
begin data.
1 1 7
1 0 3
0 1 3
0 0 7
end data.

weight by freq.

logistic regression admit
/method = enter gender.
```

Variables in the Equation

		B	S.E.	Wald	df	Sig.	Exp(B)
Step 1[a]	gender	1.695	.976	3.015	1	.082	5.444
	Constant	-.847	.690	1.508	1	.220	.429

a. Variable(s) entered on step 1: gender.

Note that Wald = 3.015 for both the coefficient for gender and for the odds ratio for gender (because the coefficient and the odds ratio are two ways of saying the same thing).

About logits

There is a direct relationship between the coefficients and the odds ratios. First, let's define what is meant by a logit: A logit is defined as the log base e (log) of the odds,

logit(p) = log(odds) = log(p/q)

Logistic regression is in reality ordinary regression using the logit as the response variable,

logit(p) = a + bX

or

log(p/q) = a + bX

**This means that the coefficients in logistic regression are in terms of
the log odds, that is, the coefficient 1.695 implies that a one unit change in
gender
results in a 1.695 unit change in the log of the odds.**

Equation can be expressed in odds by getting rid of the log. This is done by taking e to the power for both sides of the equation.

p/q = ea + bX

**The end result of all the mathematical manipulations is that the odds
ratio can be computed by raising e to the power of the logistic coefficient,**

OR = eb = e1.694596 = 5.444