

What is the Erlang Distribution?

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The Erlang Distribution is a probability distribution that is commonly used in the field of telecommunication and computer science. It is named after the Danish mathematician Agner Krarup Erlang, who developed it in the early 20th century to model the arrival and departure of telephone calls in a telephone exchange. This distribution is used to analyze and predict the behavior of systems with a large number of independent events that occur at a constant rate, such as telecommunications networks, computer systems, and queuing systems. It is a versatile and powerful tool for understanding and optimizing the performance of complex systems, making it a valuable tool for engineers and researchers in various fields.

What is the Erlang Distribution?

The Erlang distribution is a originally created by to model the number of telephone calls that an operator at a switching station may receive at once.

The distribution is used in telephone traffic engineering, queueing systems, mathematical biology, and other fields to model a variety of real-world phenomena.

Properties of the Erlang Distribution

The Erlang distribution has the following probability density function:

$$f(x; k, \mu) = \frac{x^{k-1} e^{-x/\mu}}{\mu^k (k-1)!}$$

where:

k: The shape parameter. This must be a positive integer.
 μ : The scale parameter. This must be a positive

real number.

It turns out that the Erlang distribution is a special case of the Gamma distribution when the shape parameter k is restricted to only positive real integers.

Note that the scale parameter is the reciprocal of the rate parameter, λ , i.e. $\mu = 1/\lambda$.

The Erlang distribution has the following properties:

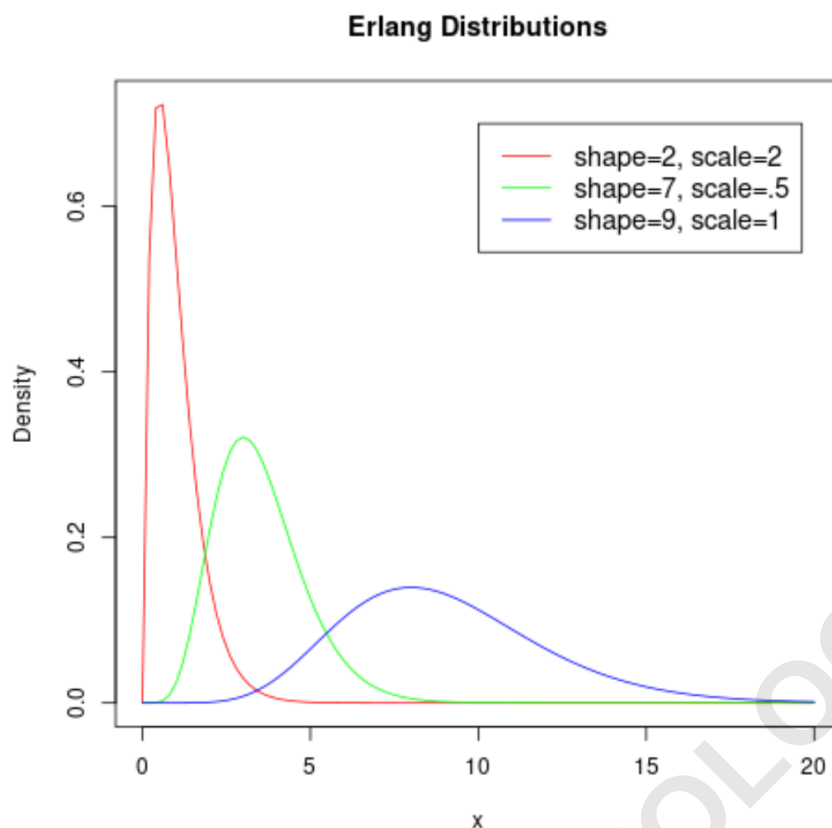
Mean: k/λ Mode: $(k-1)/\lambda$ Variance: k/λ^2 Skewness: $2/\sqrt{k}$ Kurtosis: $6/k$

The Erlang distribution has the following relationships with other distributions:

When the shape parameter, k , is equal to 1 the Erlang distribution is equal to the .When the scale parameter, μ , is equal to 2 the Erlang distribution is equal to a Chi-Squared distribution with 2 degrees of freedom.

Visualizing the Erlang Distribution

The following plot shows the shape of the Erlang distribution when it takes on different parameters:



It's interesting to see just how much the shape of the distribution changes depending on the values used for the shape and scale parameters.

Use Cases

The Erlang distribution is used in a variety of real-world settings including:

1. Call Centers

The Erlang distribution is used to model the time in between incoming calls at a call center along with the

expected number of calls.

This allows call centers to know what their staffing capacity should be during different times of the day so they can handle the incoming calls in a timely fashion without losing money by staffing too many employees during a given shift.

2. Medical Settings

The Erlang distribution is widely used to model cell cycle time distribution, which has a variety of different applications in medical settings.

3. Retail Settings

The Erlang distribution is used by retailers for modeling the frequency of interpurchase times by consumers.

This gives retailers and other businesses an idea of how often a given consumer is expected to purchase a product or service from them. This helps businesses with inventory control as well as staffing.