

# How to Easily Understand Within-Group vs. Between-Group Variation in ANOVA

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December 2, 2025

## RECOMMENDED CITATION

stats writer (2025). *How to Easily Understand Within-Group vs. Between-Group Variation in ANOVA*. PSYCHOLOGICAL SCALES. Retrieved from <https://scales.arabpsychology.com/?p=103542>

The concepts of Within-Group Variation and Between-Group Variation are fundamental to understanding Analysis of Variance (ANOVA), a powerful statistical technique used across many fields. The primary distinction lies in what each measure attempts to quantify. Within-Group Variation calculates the inherent spread or variability of scores solely within a specific treatment group, reflecting natural random fluctuations or measurement error.

In stark contrast, Between-Group Variation quantifies the differences observed among the means of two or more independent groups. Essentially, the former assesses the precision or consistency of measurements under identical conditions, while the latter determines the magnitude of the effect attributable to the different treatments or factors being tested. Analyzing the ratio of these two forms of variation allows researchers to draw robust conclusions about population differences and treatment efficacy.

## The Foundation of Analysis of Variance (ANOVA)

An ANOVA test is a statistical procedure utilized to establish whether the true means of three or more independent samples or groups are statistically equivalent. This method is crucial in experimental design when researchers need to compare the effectiveness of multiple treatments or conditions simultaneously without resorting to multiple t-tests, which would inflate the Type I error rate.

When conducting a standard one-way ANOVA, researchers formulate two essential and opposing statistical statements that guide the analysis: the null hypothesis and the alternative hypothesis. These hypotheses define the scope of the statistical inference being made:

**H<sub>0</sub> (The Null Hypothesis):** All population group means are equal ( $\mu_1 = \mu_2 = \mu_3 \dots$ ); any observed differences in the sample means are due purely to random chance.

**H<sub>A</sub> (The Alternative Hypothesis):** At least one population group mean is statistically different from the rest, suggesting a significant treatment effect has occurred.

## Decomposing Variability: Understanding the Sources of Variation

The power of ANOVA lies in its ability to partition the total variability within a dataset into components that can be attributed to specific sources. This systematic decomposition culminates in an ANOVA summary table, which provides all the necessary inputs for calculating the test statistic and drawing conclusions.

The structure of a typical ANOVA summary table highlights these critical sources of variation, organized by Sums of Squares (SS), Degrees of Freedom (df), and Mean Squares (MS):

## ANOVA

<i>Source of Variation</i>	<i>SS</i>	<i>df</i>	<i>MS</i>	<i>F</i>	<i>P-value</i>	<i>F crit</i>
Between Groups	207.2	2	103.6	7.6952	0.0023	3.3541
Within Groups	363.5	27	13.4630			
Total	570.7	29				

As indicated in the table structure, ANOVA focuses on quantifying two primary types of variation:

**Between-Group Variation (SS<sub>Between</sub>):** This is the systematic variation. It represents the total squared deviation between each specific group mean and the overall grand mean of the entire dataset. It is the component of variation that we believe is caused by the experimental treatment or factor being investigated.

**Within-Group Variation (SS<sub>Within</sub>):** This is the error or residual variation. It captures the total variation among the individual values in each group relative to that group's specific mean. This variation is considered random and uncontrollable, reflecting individual differences or measurement errors.

### The F-statistic: The Ratio of Signal to Noise

The ultimate test in ANOVA relies on the F-statistic, which acts as a ratio comparing the variation we can explain (the signal, or SS<sub>Between</sub>) against the variation we cannot explain (the noise, or SS<sub>Within</sub>). Specifically, the F-ratio is calculated as the Mean Square Between (MSB) divided by the Mean Square Within (MSW).

If the Between-Group Variation is significantly high relative to the Within-Group Variation, the resulting F-statistic will be large. A larger F-ratio implies that the differences between the group means are much greater than the differences observed within the groups themselves. This is precisely the evidence needed to reject the null hypothesis.

A high F-statistic yields a lower corresponding p-value. If this p-value falls below the predetermined significance level (e.g., 0.05), we conclude that the observed differences in group means are statistically significant and unlikely to be due to chance. The following example demonstrates how these components are calculated in practice.

### Example: Calculating Within-Group and Between-Group Variation in ANOVA

Suppose we are conducting an experiment to determine if three distinct studying methods (Method 1, Method 2, Method 3) lead to different mean exam scores. To execute this, we recruit 30

students and randomly assign 10 students to each of the three studying methods.

The raw exam scores recorded for the students in each group are detailed in the table below, forming the basis for our variation calculations:

Method 1	Method 2	Method 3
75	78	82
77	78	82
78	79	84
78	81	86
79	81	86
81	82	87
81	83	87
83	85	89
86	86	90
87	88	94

### Calculating Between-Group Variation (SS<sub>Between</sub>)

The first crucial step is quantifying the Between-Group Variation (SS<sub>Between</sub>). This calculation focuses on the squared differences between each group mean and the overall grand mean, weighted by the sample size of each group. We use the following precise formula:

$$\text{Between Group Variation (SS}_{\text{Between}}) = \sum n_j (X_j - X_{..})^2$$

The variables used in this statistical notation represent:

**$n_j$** : The sample size (number of data points) of the  $j$ th group.

**$\Sigma$** : The summation symbol, instructing us to sum the calculated values across all groups.

**$X_j$** : The calculated mean score for group  $j$ .

**$X_{..}$** : The overall grand mean, calculated from all 30 observations combined.

Before applying the formula, we must first determine the mean score for each group and the overall grand mean:

	Method 1	Method 2	Method 3
	75	78	82
	77	78	82
	78	79	84
	78	81	86
	79	81	86
	81	82	87
	81	83	87
	83	85	89
	86	86	90
	87	88	94
<b>Group Mean</b>	80.5	82.1	86.7
<b>Overall Mean</b>	83.1		

With these means calculated, we compute the total Between Group Variation:  $SS_{\text{Between}} = 10(80.5 - 83.1)^2 + 10(82.1 - 83.1)^2 + 10(86.7 - 83.1)^2 = \mathbf{207.2}$ . This value represents the variation explained by the differences in the studying methods.

### Calculating Within-Group Variation ( $SS_{\text{Within}}$ )

The second critical calculation is the Within-Group Variation ( $SS_{\text{Within}}$ ), which captures the random error variance. This is calculated by summing the squared differences between each individual observation and its respective group mean, across all groups. The formula is:

**Within Group Variation ( $SS_{\text{Within}}$ ):**  $\sum(X_{ij} - X_j)^2$

The definitions of the formula components are:

$\Sigma$ : The summation symbol, indicating the summing of squared deviations across all individual observations in all groups.

$X_{ij}$ : The  $i$ th individual observation or score within group  $j$ .

$X_j$ : The calculated mean score of group  $j$ .

We calculate the sum of squared deviations for each group based on its own mean:

**Group 1:**  $(75-80.5)^2 + (77-80.5)^2 + (78-80.5)^2 + (78-80.5)^2 + (79-80.5)^2 + (81-80.5)^2 + (81-80.5)^2 + (83-80.5)^2 + (86-80.5)^2 + (87-80.5)^2 = \mathbf{136.5}$

**Group 2:**  $(78-82.1)^2 + (78-82.1)^2 + (79-82.1)^2 + (81-82.1)^2 + (81-82.1)^2 + (82-82.1)^2 + (83-82.1)^2 + (85-82.1)^2 + (86-82.1)^2 + (88-82.1)^2 = \mathbf{104.9}$

**Group 3:**  $(82-86.7)^2 + (82-86.7)^2 + (84-86.7)^2 + (86-86.7)^2 + (86-86.7)^2 + (87-86.7)^2 + (87-86.7)^2 + (89-86.7)^2 + (90-86.7)^2 + (94-86.7)^2 = 122.1$

The final total Within Group Variation (SSWithin) is the sum of these individual group variations:  $136.5 + 104.9 + 122.1 = 363.5$ .

## Interpreting the ANOVA Results and F-statistic

When these calculated Sums of Squares (SSBetween = 207.2 and SSWithin = 363.5) are used to perform a one-way ANOVA using statistical software, the resulting output table confirms these initial calculations:

ANOVA						
<i>Source of Variation</i>	<i>SS</i>	<i>df</i>	<i>MS</i>	<i>F</i>	<i>P-value</i>	<i>F crit</i>
<b>Between Groups</b>	<b>207.2</b>	2	103.6	7.6952	0.0023	3.3541
<b>Within Groups</b>	<b>363.5</b>	27	13.4630			
Total	570.7	29				

The crucial piece of information for decision-making is the overall F-statistic (or F-ratio). This statistic, calculated by comparing the variation between groups to the variation within groups, determines the probability that the observed differences occurred by chance. The larger the F-statistic, the stronger the evidence that there is a genuine difference between the group means.

In this specific analysis, the calculated F-statistic is 7.6952, which corresponds to a p-value of **.0023**. Since this p-value is significantly smaller than the conventional significance level of 0.05, we possess sufficient evidence to reject the null hypothesis. Therefore, we conclude with confidence that the three different studying techniques do not lead to the same mean exam scores; at least one method is statistically superior or inferior to the others.

## Further Resources on Statistical Modeling

The following tutorials provide additional information and deeper insight into ANOVA models and their applications in statistical research: