

What is the difference between the Poisson and Normal distribution?

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The Poisson and Normal distributions are two commonly used statistical distributions that describe the probability of events occurring. The main difference between the two distributions lies in their shape and the type of data they are used to model. The Poisson distribution is used to model the occurrence of discrete events, such as the number of customers entering a store or the number of accidents in a given time period. It is characterized by a single parameter, the mean, which represents the average number of occurrences. The shape of the distribution is skewed and has a long tail on the right side. On the other hand, the Normal distribution is used to model continuous data, such as height or weight, and is characterized by two parameters, the mean and standard deviation. The shape of the distribution is bell-shaped and symmetrical, with most data falling within a few standard deviations of the mean. In summary, the main difference between the Poisson and Normal distributions is the type of data they are used to model and their respective shapes.

Poisson vs. Normal Distribution: What's the Difference?

The Poisson distribution and the normal distribution are two of the most commonly used probability distributions in statistics.

This tutorial provides a quick explanation of each distribution along with two key differences between the distributions.

An Overview: The Poisson Distribution

The Poisson distribution describes the probability of obtaining k successes during a given time interval.

If a random variable X follows a Poisson distribution, then the probability that $X = k$ successes can be found

by the following formula:

$$P(X=k) = \frac{\lambda^k * e^{-\lambda}}{k!}$$

where:

λ : mean number of successes that occur during a specific interval
k: number of successes
e: a constant equal to approximately 2.71828

For example, suppose a particular hospital experiences an average of 2 births per hour. We can use the formula above to determine the probability of experiencing 3 births in a given hour:

$$P(X=3) = \frac{2^3 * e^{-2}}{3!} = 0.1805$$

The probability of experiencing 3 births in a given hour is 0.1805.

An Overview: The Normal Distribution

The normal distribution describes the probability that a random variable takes on a value within a given interval.

The of a normal distribution can be written as:

$$P(X=x) = \frac{1}{\sigma\sqrt{2\pi}}e^{-1/2((x-\mu)/\sigma)^2}$$

where:

σ : Standard deviation of the distribution
 μ : Mean of the distribution
 x : Value for the random variable

If we randomly select an otter from this population, we can use the following formula to find the probability that it weighs between 38 and 42 pounds:

$$P(38 < X < 42) = \frac{1}{\sigma\sqrt{2\pi}}e^{-1/2((42-40)/5)^2} - \frac{1}{\sigma\sqrt{2\pi}}e^{-1/2((38-40)/5)^2} = 0.3108$$

The probability that the randomly selected otter weighs between 38 and 42 pounds is 0.3108.

Difference #1: Discrete vs. Continuous Data

The first difference between the Poisson and normal distribution is the type of data that each probability distribution models.

A Poisson distribution is used when you're working with discrete data that can only take on integer values equal to or greater than zero. Some examples include:

**Number of calls received per hour at a call center
Number of customers per day at a restaurant
Number of car accidents per month**

In each scenario, the random variable can only take on a value of 0, 1, 2, 3, etc.

A normal distribution is used when you're working with continuous data that can take on *any* value from negative infinity to positive infinity. Some examples include:

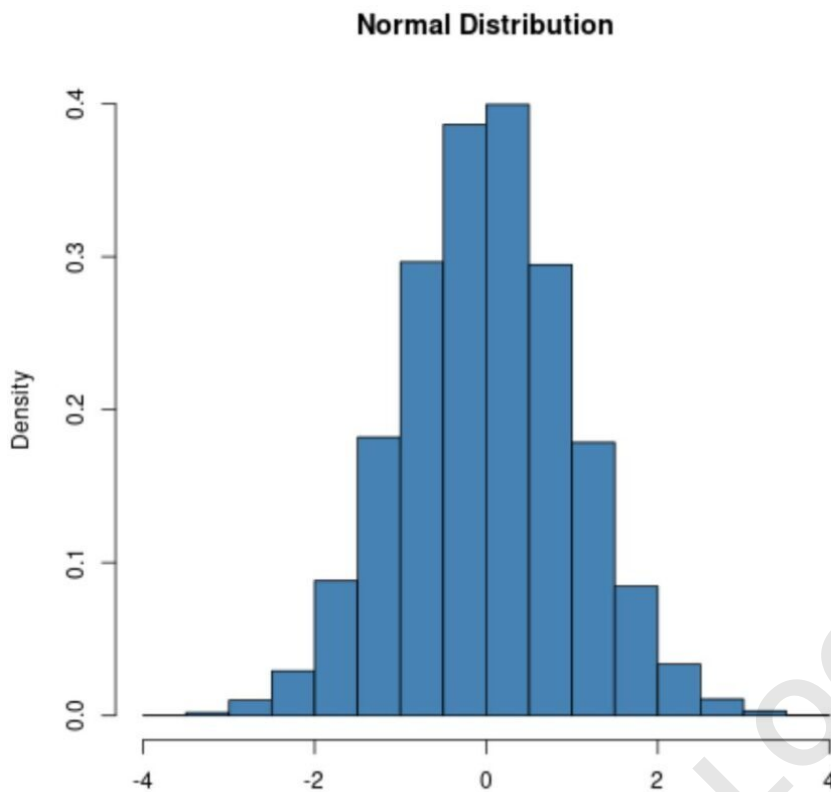
**Weight of a certain animal
Height of a certain plant
Marathon times of females
Temperature in Celsius**

In these scenarios, the random variables can take on *any* value like -11.3, 21.343435, 85, etc.

Difference #2: Shape of the Distributions

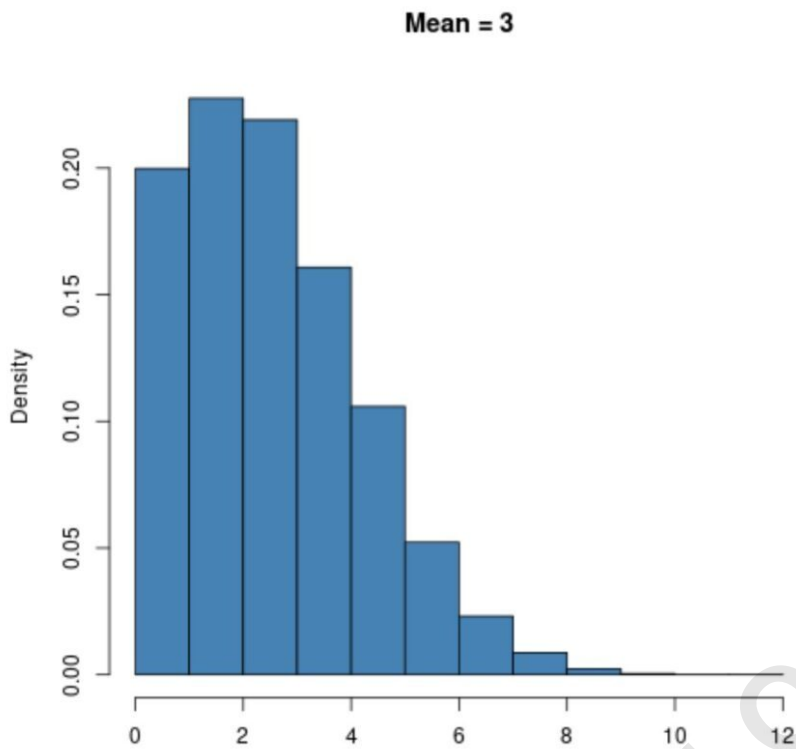
The second difference between the Poisson and normal distribution is the shape of the distributions.

A normal distribution will always exhibit a bell shape:

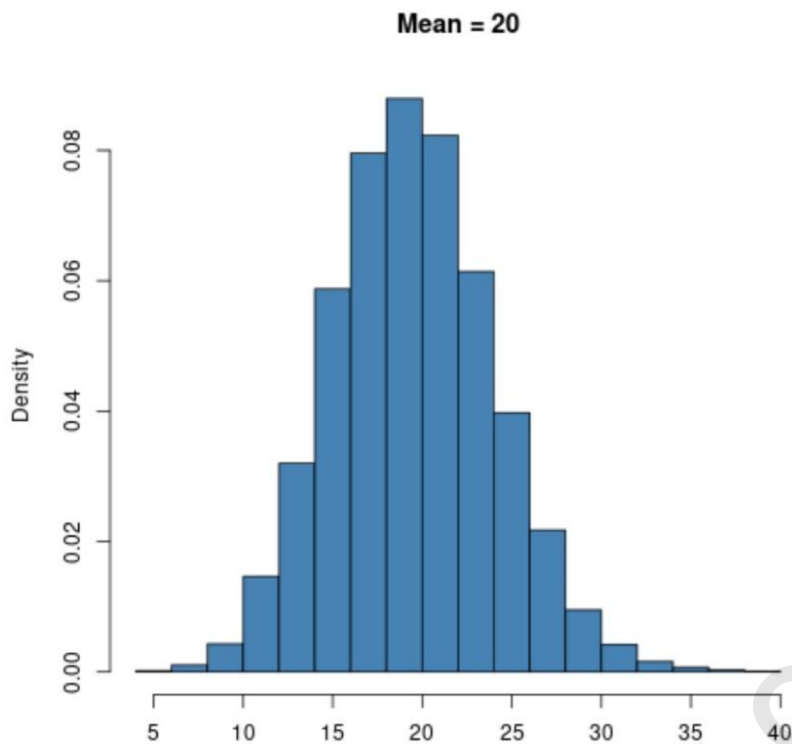


However, the shape of the Poisson distribution will vary based on the mean value of the distribution.

For example, a Poisson distribution with a small value for the mean like $\mu = 3$ will be highly :



However, a Poisson distribution with a larger value for the mean like $\mu = 20$ will exhibit a bell shape just like the normal distribution:



Notice that the lower bound for a Poisson distribution will always be zero no matter what the value for the mean is because a Poisson distribution can only be used with integer values that are equal to or greater than zero.

Additional Resources

The following tutorials provide additional information about the Poisson distribution:

The following tutorials provide additional information about the normal distribution: