What is the difference between Normal Distribution and t-Distribution?

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Normal Distribution and t-Distribution are both probability distributions commonly used in statistical analysis. However, they differ in some key aspects.

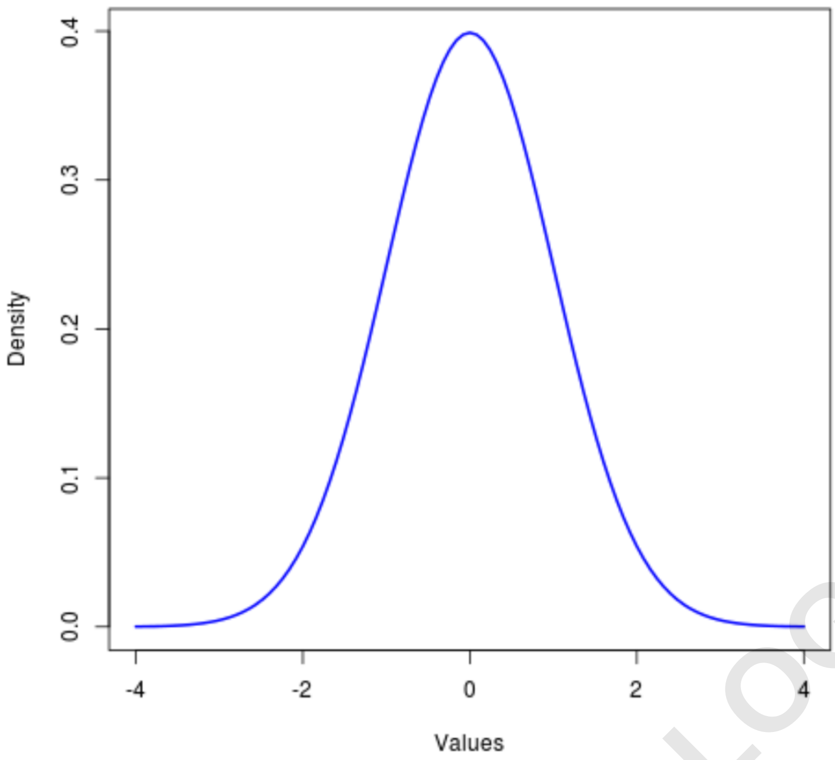
Normal Distribution, also known as the Gaussian Distribution, is a symmetric bell-shaped curve that is characterized by its mean and standard deviation. It is often used to model continuous data that is normally distributed in a population. The mean, median, and mode of a Normal Distribution are all equal, and most of the data falls within three standard deviations from the mean.

t-Distribution, also known as the Student's t-Distribution, is similar to the Normal Distribution in shape but has heavier tails, meaning it has more extreme values. It is used when the sample size is small and the population standard deviation is unknown. Unlike the Normal Distribution, the t-Distribution has an additional parameter called degrees of freedom, which affects the shape of the curve. As the sample size increases, the t-Distribution approaches the shape of the Normal Distribution.

In summary, the main difference between Normal Distribution and t-Distribution is the variability and the number of parameters used to describe the distribution. Normal Distribution is used for larger sample sizes and when the population standard deviation is known, while t-Distribution is used for smaller sample sizes and when the population standard deviation is unknown.

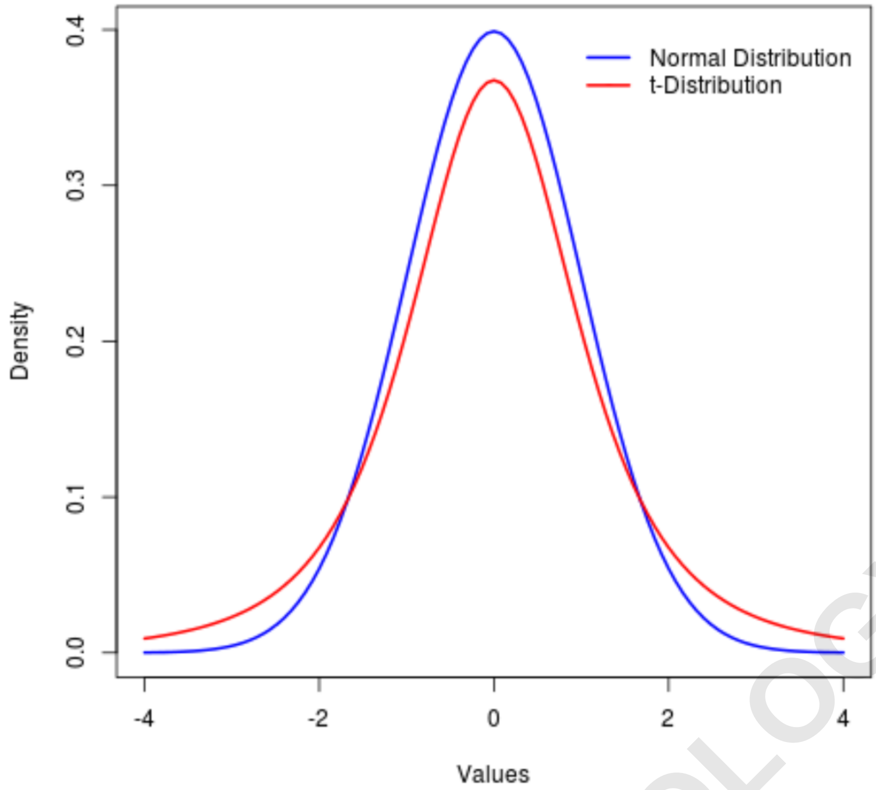
Normal Distribution vs. t-Distribution: What's the Difference?

The normal distribution is the most commonly used distribution in all of statistics and is known for being symmetrical and bell-shaped.



A closely related distribution is the t-distribution, which is also symmetrical and bell-shaped but it has heavier "tails" than the normal distribution.

That is, more values in the distribution are located in the tail ends than the center compared to the normal distribution:



In statistical jargon we use a metric called kurtosis to measure how "heavy-tailed" a distribution is. Thus, we would say that the kurtosis of a t-distribution is greater than a normal distribution.

In practice, we use the t-distribution most often when performing hypothesis tests or constructing confidence intervals.

For example, the formula to calculate a confidence interval for a population mean is as follows:

Confidence Interval = x +/- t1-α/2, n-1*(s/√n)

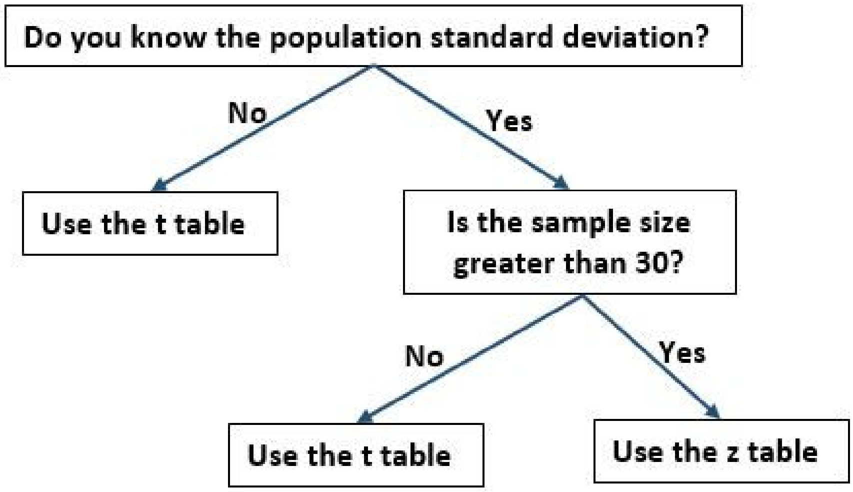
where:

x: sample meant: the critical t-value, based on the significance level α and sample size *n*s: sample standard deviationn: sample size

In this formula we use the critical value from the t table instead of the critical value from the z table when either one of the following is true:

We do not know the population standard deviation.The sample size is less than or equal to 30.

The following flow diagram provides a helpful way to know whether you should use the critical value from the t table or the z table:



The main difference between using the t-distribution compared to the normal distribution when constructing confidence intervals is that critical values from the t-distribution will be larger, which leads to *wider* confidence intervals.

For example, suppose we'd like to construct a 95% confidence interval for the mean weight for some population of turtles so we go out and collect a random sample of turtles with the following information:

Sample size n = 25Sample mean weight x = 300Sample standard deviation s = 18.5

The z-critical value for a 95% confidence level is 1.96 while a t-critical value for a 95% confidence interval with

df = 25-1 = 24 degrees of freedom is 2.0639.

Thus, a 95% confidence interval for the population mean using a z-critical value is:

95% C.I. = 300 +/- 1.96*(18.5/√25) =

While a 95% confidence interval for the population mean using a t-critical value is:

95% C.I. = 300 +/- 2.0639*(18.5/√25) =

Notice that the confidence interval with the t-critical value is wider.

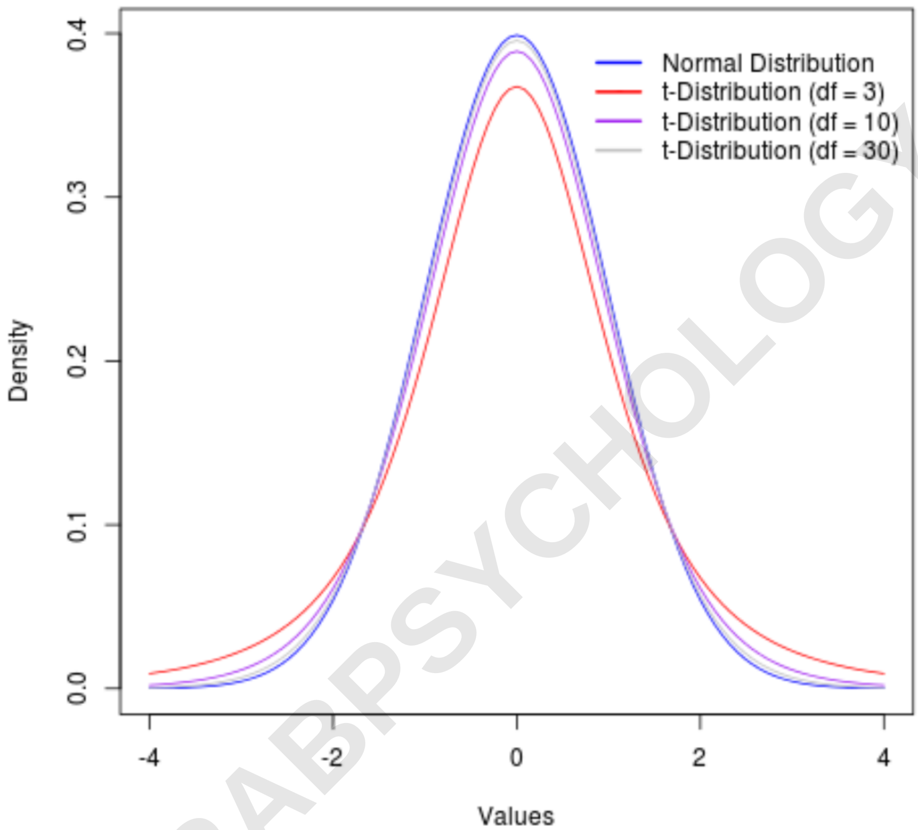
The idea here is that when we have small sample sizes, we're less certain about the true population mean so it makes since to use the t-distribution to produce wider confidence intervals that have a higher chance of containing the true population mean.

Visualizing Degrees of Freedom for the t-Distribution

It's worth noting that as the degrees of freedom increases, the t-distribution approaches the normal distribution.

To illustrate this, consider the following graph that shows the shape of the t-distribution with the following degrees of freedom:

df = 3df = 10df = 30



Beyond 30 degrees of freedom, the t-distribution and the normal distribution become so similar that the differences between using a t-critical value vs. a z-critical value in formulas becomes negligible.