

What is the difference between a normal distribution and a standard normal distribution?

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A normal distribution is a bell-shaped curve that represents a set of data with a symmetrical and unimodal distribution, where the mean, median, and mode are all equal. This type of distribution is commonly seen in natural phenomena and human behavior. On the other hand, a standard normal distribution is a specific type of normal distribution with a mean of 0 and a standard deviation of 1. It is often used in statistics as a reference point for comparing and analyzing other normal distributions. The main difference between the two is that the standard normal distribution has a standardized scale, making it easier to compare and analyze data from different normal distributions.

Normal Distribution vs. Standard Normal Distribution: The Difference

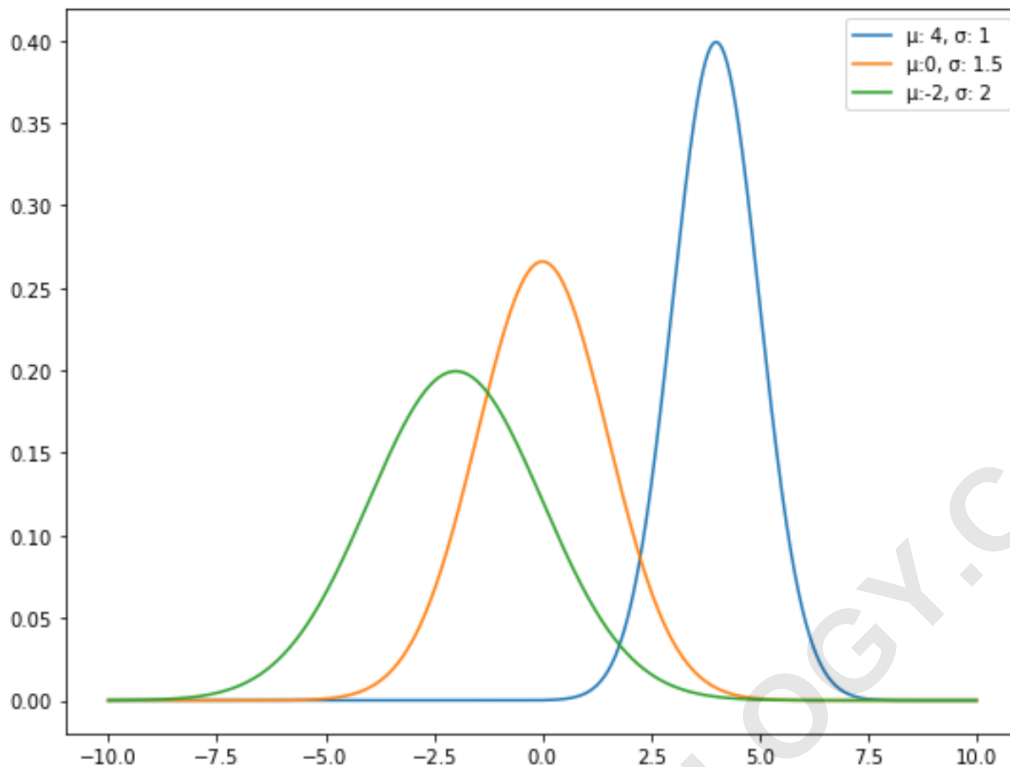
The is the most commonly used probability distribution in statistics.

It has the following properties:

Bell-shaped
Mean and median are equal; both located at the center of the distribution

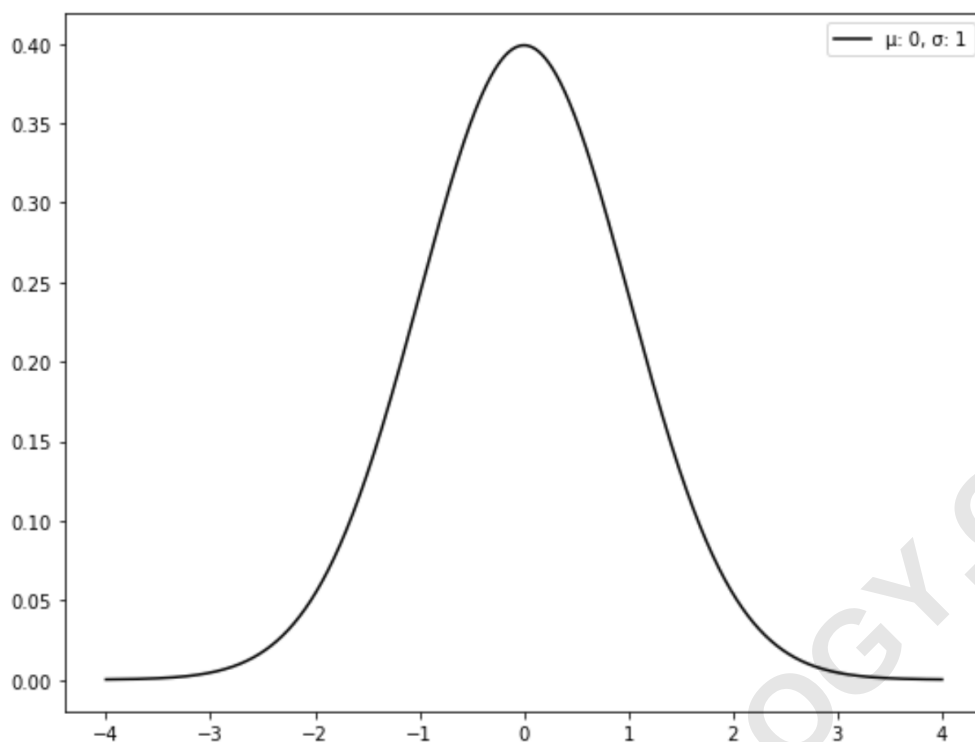
The mean of the normal distribution determines its location and the standard deviation determines its spread.

For example, the following plot shows three normal distributions with different means and standard deviations:



The standard normal distribution is a specific type of normal distribution where the mean is equal to 0 and the standard deviation is equal to 1.

The following plot shows a standard normal distribution:



How to Convert a Normal Distribution to Standard Normal Distribution

Any normal distribution can be converted into a standard normal distribution by converting the data values into z-scores, using the following formula:

$$z = (x - \mu) / \sigma$$

where:

x: Individual data value **μ :** Mean of the distribution **σ :** Standard deviation of the distribution

For example, suppose we have the following dataset

with a mean of 6 and a standard deviation of 2.152:

Data
3
4
5
6
7
8
8
8
8
8
9
9
7
6
6
5
4
4
3
2

We can convert each individual data value into a z-score by subtracting 6 from each value and dividing by 2.152:

Data	Z-Score
3	-1.39
4	-0.93
5	-0.46
6	0.00
7	0.46
8	0.93
8	0.93
8	0.93
8	0.93
8	0.93
9	1.39
9	1.39
7	0.46
6	0.00
6	0.00
5	-0.46
4	-0.93
4	-0.93
3	-1.39
2	-1.86

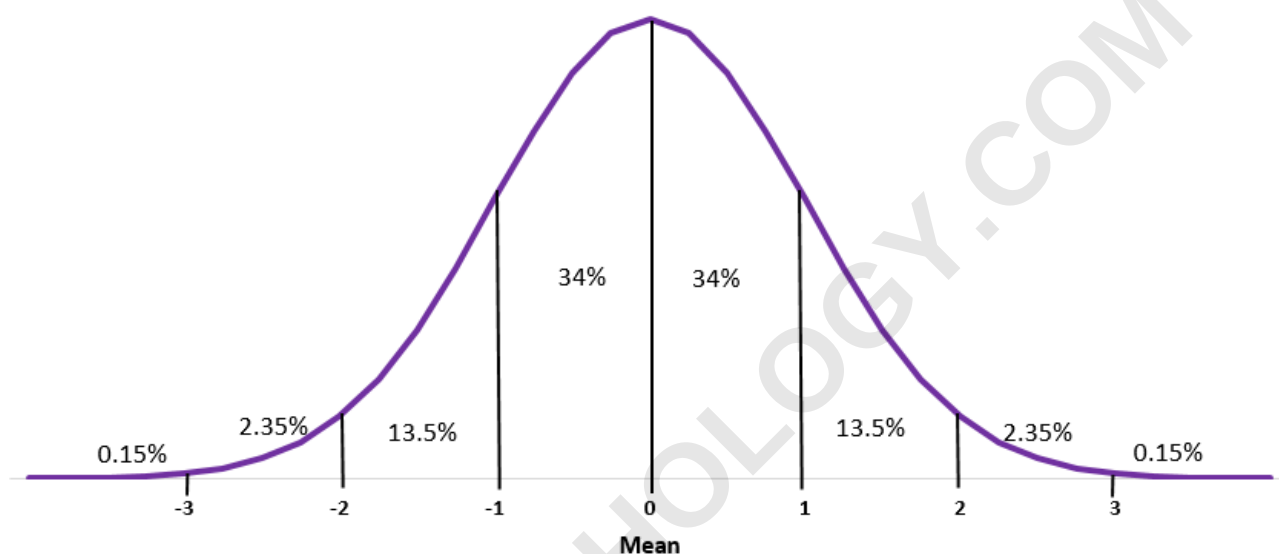
The z-score tells us how many standard deviations each data point lies from the mean. For example, the first data value of "3" lies 1.39 standard deviations below the mean.

The mean of this distribution of z-scores has a mean of zero and a standard deviation of one.

How to Use the Standard Normal Distribution

A standard normal distribution has the following properties:

About 68% of data falls within one standard deviation of the mean
About 95% of data falls within two standard deviations of the mean
About 99.7% of data falls within three standard deviations of the mean



This is known as the Empirical Rule and is used to understand the distribution of values in a dataset.

For example, suppose the height of plants in a certain garden are normally distributed with a mean of 47.4 inches and a standard deviation of 2.4 inches.

According to the Empirical Rule, what percentage of plants are less than 54.6 inches tall?

The Empirical Rule states that for a given dataset with a

normal distribution, 99.7% of data values fall within three standard deviations of the mean. This means that 49.85% of values fall between the mean and three standard deviations above the mean.

In this example, 54.6 is located three standard deviations above the mean. Since we know that 50% of data values fall below the mean in a normal distribution, a total of $50\% + 49.85\% = 99.85\%$ of values fall below 54.6.

Thus, 99.85% of plants are less than 54.6 inches tall.