What is the definition of the Law of Total Probability and what are some examples of its application?

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April 26, 2024

The Law of Total Probability is a fundamental principle in probability theory that states that the total probability of all possible outcomes in a sample space is equal to 1. This means that when considering multiple events, the sum of their probabilities must equal 1. This law is often used in probability calculations to determine the probability of a specific event occurring.

One example of the Law of Total Probability is in determining the probability of a person being left-handed. If we consider two events - being left-handed and being right-handed - the sum of their probabilities must be equal to 1. So, if the probability of being left-handed is 0.1, the probability of being right-handed must be 0.9. This principle can also be applied to more complex scenarios, such as calculating the probability of a certain disease given multiple risk factors.

In essence, the Law of Total Probability is a crucial concept in probability theory that allows us to calculate the likelihood of events occurring by considering all possible outcomes.

Law of Total Probability: Definition & Examples

In probability theory, the law of total probability is a useful way to find the probability of some event *A* when we don't directly know the probability of *A* but we do know that events *B*1, *B*2, *B*3... form a partition of the *S*.

This law states the following:

The Law of Total Probability

If *B*1, *B*2, *B*3... form a partition of the sample space *S*, then we can calculate the probability of event *A* as:

P(*A*) = ΣP(*A*|*B*i)*P(*B*i)

The easiest way to understand this law is with a simple example.

Suppose there are two bags in a box, which contain the following marbles:

Bag 1: 7 red marbles and 3 green marblesBag 2: 2 red marbles and 8 green marbles

If we randomly select one of the bags and then randomly select one marble from that bag, what is the probability that it's a green marble?

In this example, let P(*G*) = probability of choosing a green marble. This is the probability that we're interested in, but we can't compute it directly.

Instead we need to use the conditional probability of *G*, given some events *B* where the *B*i's form a partition of the sample space *S*. In this example, we have the following conditional probabilities:

P(G|B1) = 3/10 = 0.3P(G|B2) = 8/10 = 0.8

Thus, using the law of total probability we can calculate the probability of choosing a green marble as:

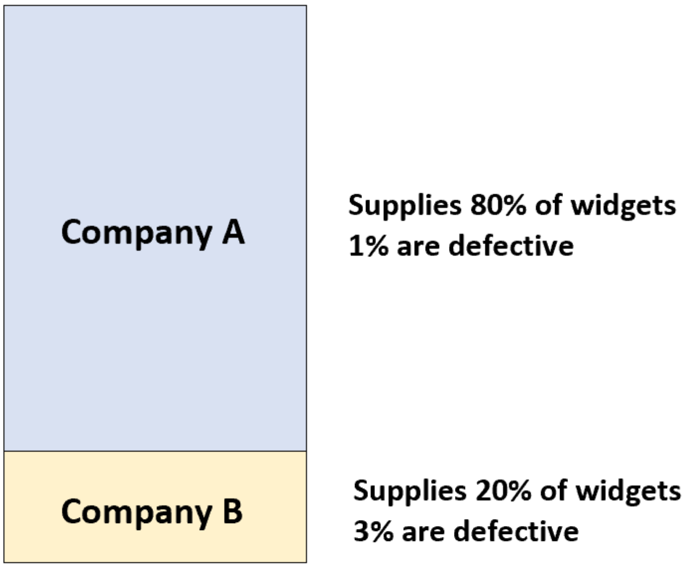
P(G) = ΣP(G|Bi)*P(Bi)P(G) = P(G|B1)*P(B1) + P(G|B2)*P(B2)P(G) = (0.3)*(0.5) + (0.8)*(0.5)P(G) = 0.55

If we randomly select one of the bags and then randomly select one marble from that bag, the probability we choose a green marble is 0.55.

Read through the next two examples to solidify your understanding of the law of total probability.

Example 1: Widgets

Company A supplies 80% of widgets for a car shop and only 1% of their widgets turn out to be defective. Company B supplies the remaining 20% of widgets for the car shop and 3% of their widgets turn out to be defective.



If we let P(*D*) = the probability of a widget being defective and *P(B*i) be the probability that the widget came from one of the companies, then we can compute the probability of buying a defective widget as:

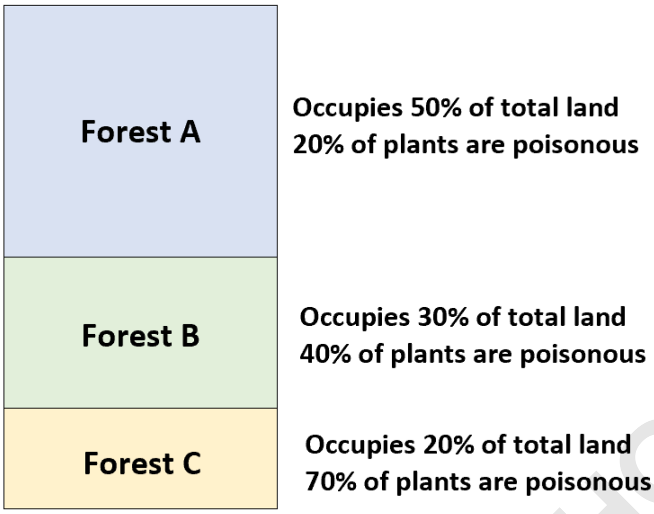
$P(D) = \Sigma P(D|Bi)*P(Bi)P(D) = P(D|B1)*P(B1) + P(D|B2)*P(B2)P(D) = (0.01)*(0.80) + (0.03)*(0.20)P(D) = 0.014$

If we randomly buy a widget from this car shop, the probability that it will be defective is 0.014.

Example 2: Forests

Forest A occupies 50% of the total land in a certain park and 20% of the plants in this forest are poisonous.

Forest B occupies 30% of the total land and 40% of the plants in it are poisonous. Forest C occupies the remaining 20% of the land and 70% of the plants in it are poisonous.



If we randomly enter this park and pick a plant from the ground, what is the probability that it will be poisonous?

If we let P(*P*) = the probability of the plant being poisonous, and *P(Bi)* be the probability that we've entered one of the three forests, then we can compute the probability of a randomly chosen plant being poisonous as:

$$P(P) = \Sigma P(P|Bi)*P(Bi)P(P) = P(P|B1)*P(B1) +$$

P(P|B2)*P(B2) + P(P|B3)*P(B3)P(P) = (0.20)*(0.50) + (0.40)*(0.30) + (0.70)*(0.20)P(P) = 0.36

If we randomly pick a plant from the ground, the probability that it will be poisonous is 0.36.