What is the definition of normal approximation to binomial and can you provide an example?

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The normal approximation to binomial is a statistical method used to estimate the probability of a certain number of successes in a fixed number of trials. It assumes that the distribution of the number of successes closely follows a normal distribution. This approximation is often used when the number of trials is large and the probability of success is not too small or too large. An example of this is flipping a fair coin 100 times and calculating the probability of getting exactly 50 heads. Using the normal approximation to binomial, we can estimate this probability by assuming a normal distribution and using the mean and standard deviation of the distribution to find the probability of getting 50 heads.

Normal Approximation to Binomial: Definition & Example

If *X* is a that follows a with *n* trials and *p* probability of success on a given trial, then we can calculate the mean (μ) and standard deviation (σ) of *X* using the following formulas:

$\mu = np\sigma = \sqrt{np(1-p)}$

It turns out that if *n* is sufficiently large then we can actually use the to approximate the probabilities related to the binomial distribution. This is known as the normal approximation to the binomial.

For *n* to be "sufficiently large" it needs to meet the following criteria:

$np \geq 5n(1-p) \geq 5$

When both criteria are met, we can use the normal distribution to answer probability questions related to the binomial distribution.

However, the normal distribution is a continuous probability distribution while the binomial distribution is a discrete probability distribution, so we must apply a continuity correction when calculating probabilities.

In simple terms, a continuity correction is the name given to adding or subtracting 0.5 to a discrete x-value.

For example, suppose we would like to find the probability that a coin lands on heads less than or equal to 45 times during 100 flips. That is, we want to find $P(X \leq 45)$. To use the normal distribution to approximate the binomial distribution, we would instead find $P(X \leq 45.5)$.

The following table shows when you should add or subtract 0.5, based on the type of probability you're trying to find:

Using Binomial Distribution	Using Normal Distribution with Continuity Correction
$X = 45$	$44.5 < X < 45.5$
$X \leq 45$	$X < 45.5$
$X < 45$	$X < 44.5$

$X \geq 45$	$X > 44.5$
$X > 45$	$X > 45.5$

The following step-by-step example shows how to use the normal distribution to approximate the binomial distribution.

Example: Normal Approximation to the Binomial

Suppose we want to know the probability that a coin lands on heads less than or equal to 43 times during 100 flips.

In this situation we have the following values:

n (number of trials) = 100X (number of successes) = 43p (probability of success on a given trial) = 0.50

To calculate the probability of the coin landing on heads less than or equal to 43 times, we can use the following steps:

First, we must verify that the following criteria are met:

$np \geq 5n(1-p) \geq 5$

In this case, we have:

np = 100*0.5 = 50n(1-p) = 100*(1 - 0.5) = 100*0.5 = 50

Both numbers are greater than 5, so we're safe to use the normal approximation.

Step 2: Determine the continuity correction to apply.

Referring to the table above, we see that we should add 0.5 when we're working with a probability in the form of X ≤ 43. Thus, we will be finding P(X< 43.5).

Step 3: Find the mean (μ) and standard deviation (σ) of the binomial distribution.

μ = n*p = 100*0.5 = 50

σ = √n*p*(1-p) = √100*.5*(1-.5) = √25 = 5

Step 4: Find the z-score using the mean and standard deviation found in the previous step.

z = (x - μ) / σ = (43.5 - 50) / 5 = -6.5 / 5 = -1.3.

Step 5: Find the probability associated with the z-score.

We can use the to find that the area under the standard normal curve to the left of -1.3 is .0968.

Thus, the probability that a coin lands on heads less than or equal to 43 times during 100 flips is .0968.

This example illustrated the following:

We had a situation where a random variable followed a binomial distribution.We wanted to find the probability of obtaining a certain value for this random variable.Since the sample size (n = 100 trials) was sufficiently large, we were able to use the normal distribution to approximate the binomial distribution.

This is a complete example of how to use the normal approximation to find probabilities related to the binomial distribution.