What is the definition of collectively exhaustive events and can you provide an example?

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Collectively exhaustive events are a set of events that include every possible outcome that could occur in a given situation. In other words, the events cover all possible outcomes and there is no room for any other potential outcomes to exist. An example of collectively exhaustive events could be a coin toss, where the two possible outcomes are "heads" and "tails". These two events cover every possible outcome of the coin toss and there is no other potential outcome that could occur. In statistics, collectively exhaustive events are important in probability calculations as they ensure that the total probability of all events occurring is equal to 1, providing a complete and accurate representation of all possible outcomes.

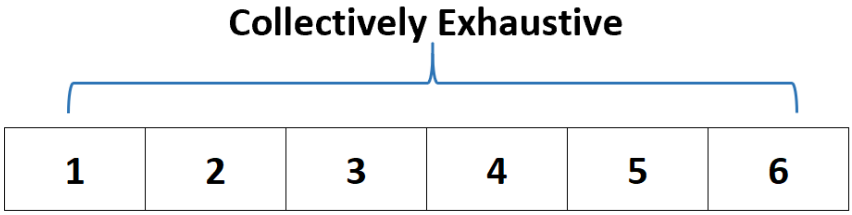
Collectively Exhaustive Events: Definition & Example

A set of events is collectively exhaustive if at least one of the events *must* occur.

For example, if we roll a die then it must land on one of the following values:

123456

Thus, we would say that the set of events {1, 2, 3, 4, 5, 6} is collectively exhaustive because the die *must* land on one of those values.



In other words, that set of events, as a *collection*,

exhausts **all possible outcomes.**

The following examples show some more situations that illustrate collectively exhaustive events:

Example 1: Flipping a Coin

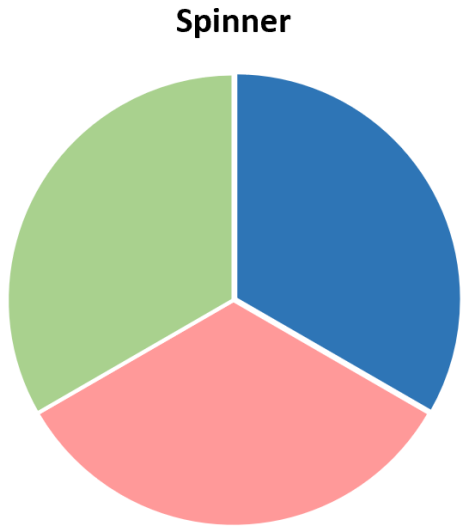
Suppose we flip a coin one time. We know that the coin must land on one of the following values:

HeadsTails

Thus, the set of events {Heads, Tails} would be collectively exhaustive.

Example 2: Spinning a Spinner

Suppose we have a spinner that has three different colors: red, blue and green.



If we spin it one time then it must land on one of the following values:

RedBlueGreen

Thus, the set of events {Red, Blue, Green} would be collectively exhaustive.

Example 3: Types of Basketball Players

Suppose we have a survey that asks individuals to select their favorite basketball player position. The only potential responses are:

Point GuardShooting GuardSmall ForwardPower ForwardCenter

Thus, the set of events {Point Guard, Shooting Guard, Small Forward, Power Forward, Center} would be collectively exhaustive.

However, the set of events {Point Guard, Shooting Guard, Small Forward} would *not* be collectively exhaustive because it does not contain all possible outcomes.

The Importance of Collectively Exhaustive Events in Surveys

When designing surveys, it's particularly important that the responses to the questions are collectively exhaustive.

For example, suppose a survey asks the following question:

What is you favorite basketball player position?

And suppose the potential responses were:

Point GuardShooting GuardSmall ForwardPower Forward

Since the position *Center* was left out, these responses are not collectively exhaustive.

This means that someone who prefers *Center* as their favorite position will have to pick one of the other options, which means the responses to the survey won't reflect the true opinions of the respondents.

Collectively Exhaustive vs. Mutually Exclusive

Events are if they cannot occur at the same time.

For example, let event A be the event that a die lands on an even number and let event B be the event that a die lands on an odd number.

We would define the for the events as follows:

A = {2, 4, 6}B = {1, 3, 5}

Notice that there is no overlap between the two sample spaces, which means they're mutually exclusive. They also happen to be collectively exhaustive because combined they're able to account for all the potential outcomes of the die roll.

However, suppose we define event A and event B as follows:

A = {1, 2, 3, 4}B = {3, 4, 5, 6}

In this case, there is some overlap between A and B so they are not mutually exclusive. However, combined they're still able to account for all the potential outcomes of the die roll.

This illustrates an important point: A set of events can be collectively exhaustive without being mutually exclusive.