What is the definition and example of the Satterthwaite Approximation?

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The Satterthwaite Approximation is a mathematical method used to estimate the degrees of freedom in a statistical test when the sample sizes are unequal. It is typically used in situations where the traditional t-test cannot be applied due to unequal variances or unequal sample sizes. The approximation calculates a more accurate estimate of the degrees of freedom, which is then used to calculate the p-value for the test. An example of the Satterthwaite Approximation would be using it to compare the mean test scores of two groups with unequal sample sizes and unequal variances.

The Satterthwaite Approximation: Definition & Example

The Satterthwaite approximation is a formula used to find the "effective degrees of freedom" in a two-sample t-test.

It used most commonly in , which compares the means of two independent samples without assuming that the populations the came from have equal variances.

The formula for the Satterthwaite approximation is as follows:

Degrees of freedom: (s12/n1 + s22/n2)2 / { + }

where:

s12, s22: The sample variance of the first and second sample, respectively.n1, n2: The sample size of the first and second sample, respectively.

The following example shows how to use the Satterthwaite approximation to calculate the effective degrees of freedom.

Example: Calculating the Satterthwaite approximation

Suppose we want to know if the mean height of two different plant species is equal so we go out and collect two simple random samples of each species and measure the height of the plants in each sample.

The following values show the height (in inches) for each sample:

Sample 1: 14, 15, 15, 15, 16, 18, 22, 23, 24, 25, 25

Sample 2: 10, 12, 14, 15, 18, 22, 24, 27, 31, 33, 34, 34, 34

The means, variances, and sample sizes turn out to be:

x1= 19.27x2 = 23.69s12 = 20.42s22= 83.23n1 = 11n2=13

Next, we can plug in the values for the variances and sample sizes into the Satterthwaite approximation formula to find the effective degrees of freedom:

df = (s12/n1 + s22/n2)2 / { + }

df = (20.42/11 + 83.23/13)2/{ + } = 18.137

The effective degrees of freedom turns out to be 18.137.

Lastly, would will find the *t* critical value in the t-distribution table that corresponds to a two-tailed test with alpha = .05 for 18 degrees of freedom:

	P						
one-tail	0.1	0.05	0.025	0.01	0.005	0.001	0.0005
two-tails	0.2	0.1	0.05	0.02	0.01	0.002	0.001
DF							
1	3.078	6.314	12.706	31.821	63.656	318.289	636.578
2	1.886	2.92	4.303	6.965	9.925	22.328	31.6
3	1.638	2.353	3.182	4.541	5.841	10.214	12.924
4	1.533	2.132	2.776	3.747	4.604	7.173	8.61
5	1.476	2.015	2.571	3.365	4.032	5.894	6.869
6	1.44	1.943	2.447	3.143	3.707	5.208	5.959
7	1.415	1.895	2.365	2.998	3.499	4.785	5.408
8	1.397	1.86	2.306	2.896	3.355	4.501	5.041
9	1.383	1.833	2.262	2.821	3.25	4.297	4.781
10	1.372	1.812	2.228	2.764	3.169	4.144	4.587
11	1.363	1.796	2.201	2.718	3.106	4.025	4.437
12	1.356	1.782	2.179	2.681	3.055	3.93	4.318
13	1.35	1.771	2.16	2.65	3.012	3.852	4.221
14	1.345	1.761	2.145	2.624	2.977	3.787	4.14
15	1.341	1.753	2.131	2.602	2.947	3.733	4.073
16	1.337	1.746	2.12	2.583	2.921	3.686	4.015
17	1.333	1.74	2.11	2.567	2.898	3.646	3.965
18	1.33	1.734	2.101	2.552	2.878	3.61	3.922
19	1.328	1.729	2.093	2.539	2.861	3.579	3.883
20	1.325	1.725	2.086	2.528	2.845	3.552	3.85
21	1.323	1.721	2.08	2.518	2.831	3.527	3.819
22	1.321	1.717	2.074	2.508	2.819	3.505	3.792
23	1.319	1.714	2.069	2.5	2.807	3.485	3.768
24	1.318	1.711	2.064	2.492	2.797	3.467	3.745
25	1.316	1.708	2.06	2.485	2.787	3.45	3.725
26	1.315	1.706	2.056	2.479	2.779	3.435	3.707
27	1.314	1.703	2.052	2.473	2.771	3.421	3.689
28	1.313	1.701	2.048	2.467	2.763	3.408	3.674
29	1.311	1.699	2.045	2.462	2.756	3.396	3.66
30	1.31	1.697	2.042	2.457	2.75	3.385	3.646
60	1.296	1.671	2	2.39	2.66	3.232	3.46
120	1.289	1.658	1.98	2.358	2.617	3.16	3.373
1000	1.282	1.646	1.962	2.33	2.581	3.098	3.3
Inf	1.282	1.645	1.96	2.326	2.576	3.091	3.291

The t critical value is 2.101.

We would then calculate our test statistic to be:

Test statistic: (x1 - x2) / (√s12/n1 + s22/n2)

Test statistic: (19.27 - 23.69) / (√20.42/11 + 83.23/13) =

-4.42 / 2.873 = -1.538

Since the absolute value of our test statistic (1.538) is not larger than the t critical value, we fail to reject the null hypothesis of the test.

There is not sufficient evidence to say that the means of the two populations are significantly different.

The Satterthwaite Approximation in Practice

In practice, you will rarely have to calculate the Satterthwaite approximation by hand.

Instead, common statistical software like R, Python, Excel, SAS, and Stata can all use the Satterthwaite approximation to calculate the effective degrees of freedom automatically for you.