

What is the calculation for determining the degrees of freedom in any t-test?

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June 26, 2024

RECOMMENDED CITATION

stats writer (2024). *What is the calculation for determining the degrees of freedom in any t-test?*. PSYCHOLOGICAL SCALES. Retrieved from <https://scales.arabpsychology.com/?p=153698>

The degrees of freedom in a t-test is a mathematical concept used to determine the number of independent pieces of information available in a sample. It is calculated by subtracting the number of groups being compared from the total sample size. This calculation is important because it helps determine the critical value of t , which is necessary for determining the significance of the test results. In statistical analysis, having a higher number of degrees of freedom increases the accuracy and reliability of the results. Therefore, the calculation for degrees of freedom plays a crucial role in any t-test and should be carefully considered in order to obtain accurate and meaningful results.

Calculate Degrees of Freedom for Any T-Test

In statistics, there are three commonly used t-tests:

: Used to compare a population mean to some value.

: Used to compare two population means.

: Used to compare two population means when each observation in one sample can be paired with an observation in the other sample.

When performing each t-test, you'll have to calculate a test statistic and a corresponding degrees of freedom.

Here is how to calculate the degrees of freedom for each type of test:

One Sample t-test: $df = n - 1$ where n is the total number of observations.

Two Sample t-test: $df = n1 + n2 - 2$ where $n1$, $n2$ are the total observations from each sample.

Paired Samples t-test: $n-1$ where n is the total number of pairs.

The following examples show how to calculate the degrees of freedom for each type of t-test in practice.

Example 1: Degrees of Freedom for One Sample t-test

Suppose we want to know whether or not the mean weight of a certain species of turtle is equal to 310 pounds.

Suppose we collect a random sample of turtles with the following information:

Sample size $n = 40$ Sample mean weight $\bar{x} = 300$ Sample standard deviation $s = 18.5$

We will perform a one sample t-test with the following hypotheses:

$H_0: \mu = 310$ (population mean is equal to 310 pounds)
 $H_A: \mu \neq 310$ (population mean is not equal to 310 pounds)

First, we'll calculate the test statistic:

Next, we'll calculate the degrees of freedom:

$$df = n - 1 = 40 - 1 = 39$$

Lastly, we'll plug in the test statistic and degrees of freedom into the to find that the p-value is 0.00149.

Since this p-value is less than our significance level $\alpha = 0.05$, we reject the null hypothesis. We have sufficient evidence to say that the mean weight of this species of turtle is not equal to 310 pounds.

Example 2: Degrees of Freedom for Two Sample t-test

Suppose we want to know whether or not the mean weight between two different species of turtles is equal.

Suppose we collect a random sample of turtles from each population with the following information:

Sample 1:

Sample size $n_1 = 40$ Sample mean weight $\bar{x}_1 = 300$ Sample standard deviation $s_1 = 18.5$

Sample 2:

Sample size $n_2 = 38$ Sample mean weight $\bar{x}_2 = 305$ Sample standard deviation $s_2 = 16.7$

We will perform a two sample t-test with the following hypotheses:

$H_0: \mu_1 = \mu_2$ (the two population means are equal) $H_A: \mu_1 \neq \mu_2$ (the two population means are not equal)

First, we will calculate the pooled standard deviation s_p :

$$s_p = \sqrt{(n_1-1)s_1^2 + (n_2-1)s_2^2 / (n_1+n_2-2)} = \sqrt{(40-1)18.52 + (38-1)16.72 / (40+38-2)} = 17.647$$

Next, we will calculate the test statistic t :

$$t = (\bar{x}_1 - \bar{x}_2) / s_p(\sqrt{1/n_1 + 1/n_2}) = (300-305) / 17.647(\sqrt{1/40 + 1/38}) = -1.2508$$

Next, we'll calculate the degrees of freedom:

$$df = n_1 + n_2 - 2 = 40 + 38 - 2 = 76$$

Lastly, we'll plug in the test statistic and degrees of

freedom into the to find that the p-value is 0.21484.

Since this p-value is not less than our significance level $\alpha = 0.05$, we fail to reject the null hypothesis. We do not have sufficient evidence to say that the mean weight of turtles between these two populations is different.

Example 3: Degrees of Freedom for Paired Samples t-test

Suppose we want to know whether or not a certain training program is able to increase the max vertical jump (in inches) of college basketball players.

To test this, we may recruit a of 20 college basketball players and measure each of their max vertical jumps. Then, we may have each player use the training program for one month and then measure their max vertical jump again at the end of the month.

Player	Max Vertical Jump Before Training Program	Max Vertical Jump After Training Program
Player 1	22	24
Player 2	20	22
Player 3	19	19
Player 4	24	22
Player 5	25	28
Player 6	25	26
Player 7	28	28
Player 8	22	24
Player 9	30	30
Player 10	27	29
Player 11	24	25
Player 12	18	20
Player 13	16	17
Player 14	19	18
Player 15	19	18
Player 16	28	28
Player 17	24	26
Player 18	25	27
Player 19	25	27
Player 20	23	24

To determine whether or not the training program actually had an effect on max vertical jump, we will perform a paired samples t-test.

First, we'll calculate the following summary data for the differences:

$\bar{x}_{diff}$: sample mean of the differences = -0.95
 s_{diff} : sample standard deviation of the differences = 1.317
 n : sample size (i.e. number of pairs) = 20

We will perform a paired samples t-test with the following hypotheses:

$H_0: \mu_1 = \mu_2$ (the two population means are equal) $H_A: \mu_1 \neq \mu_2$ (the two population means are not equal)

Next, we'll calculate the test statistic:

$$t = x_{\text{diff}} / (s_{\text{diff}} / \sqrt{n}) = -0.95 / (1.317 / \sqrt{20}) = -3.226$$

Next, we'll calculate the degrees of freedom:

$$df = n - 1 = 20 - 1 = 19$$

According to the , the p-value associated with $t = -3.226$ and degrees of freedom $= n - 1 = 20 - 1 = 19$ is 0.00445.

Since this p-value is less than our significance level $\alpha = 0.05$, we reject the null hypothesis. We have sufficient evidence to say that the mean max vertical jump of players is different before and after participating in the training program.

The following calculators can be used to automatically perform t-tests based on data that you provide: