What is the Breusch-Pagan test?

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The Breusch-Pagan test is a fundamental diagnostic tool in econometrics and statistics, specifically designed to evaluate whether the assumption of constant variance in the error terms of a linear regression model is upheld. This condition, known as homoscedasticity, is critical for ensuring that parameter estimates derived from Ordinary Least Squares (OLS) regression are efficient and that hypothesis tests are valid. If this assumption is violated--a state known as heteroskedasticity--the standard errors become unreliable, potentially leading to inaccurate inferences and misleading conclusions regarding the significance of predictor variables. The Breusch-Pagan test offers a formal, statistically rigorous method to detect this deviation, relying on the analysis of the squared residuals derived from the initial regression.

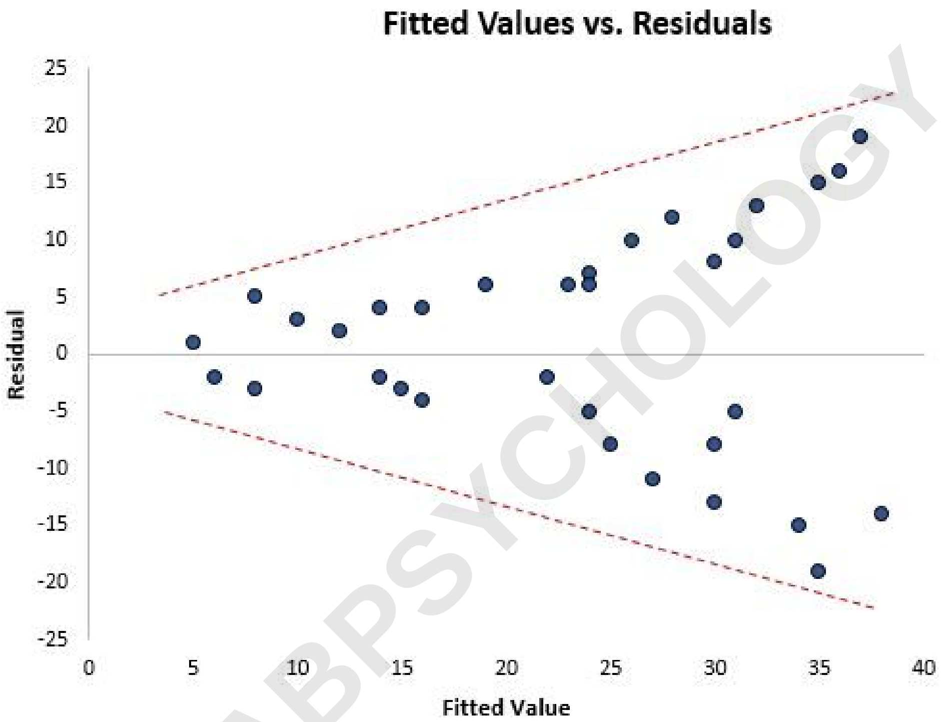
Historically, the development of this test by Trevor Breusch and Adrian Pagan provided a significant advancement over purely visual checks for variance instability. The core mechanism involves assessing the relationship between the squared residuals and the predictor variables or fitted values of the model. By transforming the problem of variance constancy into a test of linearity, the Breusch-Pagan procedure yields a powerful Chi-Square test statistic that allows researchers to formally reject the null hypothesis of constant variance when sufficient evidence of non-constant variance exists. Understanding and properly applying this diagnostic test is mandatory for any thorough regression analysis to ensure the integrity and robustness of the findings.

One of the key assumptions underlying classical linear regression models (CLRM) is that the variance of the error term, or residuals, remains constant across all levels of the independent variables. This crucial property is defined as **homoscedasticity**. When this assumption holds true, the OLS estimator remains unbiased and consistent, and its efficiency is maximized among all linear unbiased estimators. This statistical environment allows for the most reliable calculation of standard errors, which are vital for constructing accurate confidence intervals and performing precise significance tests on the regression coefficients.

Conversely, when this essential assumption of equal variance is violated, the model suffers from **heteroscedasticity**. This condition implies that the spread of the residuals is systematically related to the magnitude of one or more predictor variables or the predicted outcome itself. For instance, in economic models, the variance of income might increase as average spending increases. When heteroskedasticity is present, the OLS estimates of the coefficients remain unbiased, but the formulas used to calculate the standard errors become invalid. Consequently, t-statistics and F-statistics can be severely distorted, leading to incorrect inferences, such as falsely declaring a variable statistically significant when it is not, or vice versa.

While formal statistical tests are necessary for definitive conclusions, an initial step in diagnosing potential variance issues involves creating a diagnostic plot of the residuals against the fitted values of the regression model. This visual inspection provides immediate insight into the distribution pattern of the errors. Ideally, under **homoscedasticity**, the scatter plot should resemble a random cloud of points centered around zero, without any discernible shape or pattern, indicating uniform variance across the range of fitted values.

If, however, the residuals display a systematic pattern--such as a conical shape where the spread (variance) increases as the fitted values increase, or perhaps decreases--it serves as a powerful visual indication that **heteroscedasticity** is present. This widening or narrowing pattern signals that the error variance is not constant, thereby violating the fundamental assumption required for efficient OLS estimation. The following image provides a classic illustration of residuals exhibiting non-constant variance.



To move beyond subjective visual assessment and establish a formal diagnosis of non-constant variance, the Breusch-Pagan test (BP Test) provides the necessary statistical rigor. This tutorial will offer a comprehensive explanation of the theoretical framework and implementation of the BP test, followed by a step-by-step numerical example demonstrating how the results are derived and interpreted.

The Theoretical Framework and Hypotheses of the Breusch-Pagan Test

The Breusch-Pagan test operates by establishing a set of formal hypotheses regarding the variance structure of the error terms in the regression model. The objective is to determine if the variance of the squared residuals can be explained by the predictor variables used in the primary regression. If the predictor variables significantly explain the variation in the squared residuals, it suggests that the variance is non-constant, thereby confirming the presence of heteroskedasticity.

The test structure is built around the following competing statistical hypotheses, which are standard for diagnostics seeking to prove the presence of an issue (heteroscedasticity) against the ideal default state (homoscedasticity):

Null Hypothesis (H0): Homoscedasticity is present (The variance of the residuals is constant across all observations, implying that the error terms are distributed with equal variance)
Alternative Hypothesis (HA): Heteroscedasticity is present (The variance of the residuals is not constant, meaning the error terms are not distributed with equal variance, and variance is related to the explanatory variables)

The decision rule for the test is based on comparing the calculated p-value to a predetermined significance level, often denoted as alpha (α). Typically, researchers select $\alpha = 0.05$. If the calculated p-value derived from the test is less than this chosen significance level ($p < \alpha$), then the evidence is deemed strong enough to reject the null hypothesis (H0). Rejecting H0 leads to the conclusion that the error variance is non-constant, and therefore, **heteroscedasticity** is present in the regression model. This outcome signals that robust standard error estimation methods should be employed for reliable inference.

Step-by-Step Procedure for Implementing the Breusch-Pagan Test

Although modern statistical software automates the calculations, understanding the underlying steps of the Breusch-Pagan procedure is essential for proper interpretation. The process involves two distinct regression stages, transforming the original problem into a test of explained variation in the squared errors. The core steps are detailed below, emphasizing the sequence required to generate the necessary test statistic:

Fit the Primary Regression Model: Begin by fitting the original linear regression model using OLS. This step yields the estimated coefficients and, more importantly for this test, the residuals (the differences between the observed and predicted response values).
Calculate the Squared Residuals: Obtain the residuals from the initial model and square each one. These squared residuals, denoted as e_i^2, serve as proxies for the variance of the error term at each observation point.
Fit the Auxiliary Regression Model: Construct and fit a new, auxiliary regression model. In this new model, the squared residuals (e_i^2) calculated in Step 2 are used as the response (dependent) variable, and the original predictor variables ($X_1, X_2, \dots$) are used as the explanatory variables. The goal of this auxiliary regression is to determine if the predictor variables can systematically explain the variation in the error variance.
Calculate the Chi-Square test statistic: The final crucial step involves calculating the BP test

statistic, $BP = n cdot R^2_{new}$. This statistic is derived from the auxiliary regression results, where:

n: Represents the total number of observations (data points) in the dataset.
R2new: Represents the Coefficient of Determination (R-squared value) obtained from the new regression model that used the squared residuals as the response values.

The resulting BP test statistic, BP, follows a Chi-Square test statistic distribution with degrees of freedom equal to p, where p is the number of predictors (excluding the intercept) in the original regression model. If this calculated statistic exceeds the critical Chi-Square value corresponding to the chosen significance level, or if the associated p-value is less than some significance level (i.e. α = .05) then reject the null hypothesis and conclude that heteroscedasticity is present. Otherwise, fail to reject the null hypothesis. In this case, it's assumed that homoscedasticity is present. Note that most statistical software can easily perform the Breusch-Pagan test so you will likely never have to perform these steps by hand, but it's useful to know what's going on behind the scenes.

Case Study: Applying the Breusch-Pagan Test to Player Ratings Data

To illustrate the methodology, consider a dataset containing metrics for ten basketball players. We aim to model a player's performance rating based on their game statistics: points, assists, and rebounds. Before relying on the standard errors produced by the OLS estimation, we must verify the assumption of homoscedasticity using the Breusch-Pagan procedure. The raw data used for the primary regression is presented below:

rating	points	assists	rebounds
90	25	5	11
85	20	7	8
82	14	7	10
88	16	8	6
94	27	5	6
90	20	7	9
76	12	6	6
75	15	9	10
87	14	9	10
86	19	5	7

Using statistical software, we fit the following linear regression equation:

rating = 62.47 + 1.12*(points) + 0.88*(assists) - 0.43*(rebounds)

We then use this model to make predictions for the rating of each player and calculated the squared residuals (i.e. the squared difference between the predicted rating and the actual rating). These squared errors are essential for the second stage of the test, serving as the response variable in the auxiliary regression:

rating	points	assists	rebounds	predicted rating	squared residuals
90	25	5	11	90.17	0.03
85	20	7	8	87.62	6.86
82	14	7	10	80.05	3.81
88	16	8	6	84.88	9.73
94	27	5	6	94.54	0.30
90	20	7	9	87.19	7.88
76	12	6	6	78.64	6.96
75	15	9	10	82.93	62.96
87	14	9	10	81.82	26.88
86	19	5	7	85.16	0.70

Calculating and Interpreting the Breusch-Pagan Test Statistic

Next, we fit a new regression model using the squared residuals as the response values and the original predictor variables (points, assists, rebounds) as the explanatory variables once again. From this auxiliary regression, we extract the total sample size (n) and the R-squared value (R^2_{new}). We find the following key metrics:

n: 10
R2new: 0.600395

Thus, our Chi-Square test statistic for the Breusch-Pagan test is calculated as $BP = n \cdot R^2_{new} = 10 \times 0.600395 = \mathbf{6.00395}$. The degrees of freedom for this test are determined by the number of predictor variables, $p = 3$.

According to the Chi-Square distribution, the p-value that corresponds to $X^2 = 6.00395$ with 3 degrees of freedom is calculated to be **0.111418**. We set our significance level at $\alpha = 0.05$. Since this p-value (0.111418) is not less than the chosen significance level (0.05), we fail to reject the null hypothesis. Thus, we assume that homoscedasticity is present in the regression model, and there is no statistically significant evidence of heteroskedasticity.

Conclusion and Practical Recommendations

The Breusch-Pagan test is an indispensable tool in the diagnostic phase of regression analysis, ensuring that the model adheres to the fundamental assumption of constant error variance. While the calculations demonstrated here provide a clear understanding of the test's mechanism, professional practice dictates the use of specialized statistical software for efficiency and precision.

When the BP test results in a rejection of the null hypothesis, confirming heteroscedasticity, researchers must take corrective action. The most common remedies involve using alternative estimation techniques, such as Weighted Least Squares (WLS), or applying robust standard errors (like White's standard errors) to the OLS results. Robust standard errors correct the standard error estimates without changing the coefficient estimates, thus allowing for valid hypothesis testing even when heteroscedasticity is present. The following tutorials provide step-by-step examples of how to perform the Breusch-Pagan test in different statistical programs: