What is the annotated output for Canonical Correlation Analysis in SAS?

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Canonical Correlation Analysis (CCA) is a statistical technique used to analyze the relationship between two sets of variables. The annotated output for CCA in SAS provides a detailed summary of the results obtained from the analysis. It includes information such as the correlation coefficients, eigenvalues, and canonical loadings, which help to understand the strength and direction of the relationship between the two sets of variables. Additionally, the output also presents graphical representations, such as scatter plots and biplots, to aid in visualizing the relationship between the variables. The annotated output for CCA in SAS serves as a comprehensive guide for interpreting the results of the analysis and can be used to make informed decisions based on the findings.

Canonical Correlation Analysis | SAS Annotated Output

This page shows an example of a canonical correlation analysis in SAS with footnotes explaining the output. A researcher has collected data on three psychological variables, four academic variables (standardized test scores) and gender for 600 college freshman. She is interested in how the set of psychological variables relates to the academic variables and gender. In particular, the researcher is interested in how many dimensions are necessary to understand the association between the two sets of variables.

We have a data file,

https://stats.idre.ucla.edu/wp-content/uploads/2016/02/mmr.sas7bdat, with 600 observations on eight variables. The psychological variables are locus of control, self-concept and motivation. The academic variables are standardized test scores in reading, writing, math and science. Additionally, the variable female is a zero-one indicator variable with the one indicating a female student. The researcher is interested in the relationship between the psychological variables and the academic variables, with gender considered as well. Canonical correlation analysis aims to find pairs of linear combinations of each group of variables that are highly correlated. These linear combinations are called canonical variates. Each canonical variate is orthogonal to the other canonical variates except for the one with which its correlation has been maximized. The possible number of such pairs is limited to the number of variables in the

smallest group. In our
example, there are three psychological variables and more than three academic
variables. Thus, a canonical correlation analysis on these sets of variables
will generate three pairs of canonical variates.

To begin, let's read in and explore the dataset.

proc means data = mmr; run;

The SAS System

The MEANS Procedure

Variable Label N Mean Std Dev Minimum Maximum
ID 600 300.5000000 173.3493582 1.0000000 600.0000000
LOCUS_OF_CONTROL locus of control 600 0.0965333 0.6702799 -2.2300000 1.3600000
SELF_CONCEPT self-concept 600 0.0049167 0.7055125 -2.6199999 1.1900001
MOTIVATION motivation 600 0.6608333 0.3427294 0 1.0000000
READ reading score 600 51.9018334 10.1029830 28.2999992 76.0000000

WRITE writing score 600 52.3848333 9.7264550 25.5000000 67.0999985
MATH math score 600 51.8490000 9.4147363 31.7999992 75.5000000
SCIENCE science score 600 51.7633332 9.7061789 26.0000000 74.1999969
FEMALE 600 0.5450000 0.4983864 0 1.0000000

To run our canonical correlation, we will use the cancorr procedure in
SAS. We list the set of variables in our first group in the var
statement and the set of
variables in our second group in the with statement. We include the
optional commands vprefix, wprefix, vname and wname in the
proc cancor statement to give
understandable
prefixes to our sets of variables and make the output easier to interpret.

proc cancorr data=mmr
vprefix=Psych vname='Psychological Measurements'

wprefix=Academic wname='Academic Measurements';
var locus_of_control self_concept motivation;
with read write math science female;
run;

.....

Correlations Among the Original Variables

Correlations Among the Psychological Measurements

LOCUS_OF_
CONTROL SELF_CONCEPT MOTIVATION

LOCUS_OF_CONTROL 1.0000 0.1712 0.2451
SELF_CONCEPT 0.1712 1.0000 0.2886
MOTIVATION 0.2451 0.2886 1.0000

Correlations Among the Academic Measurements

READ WRITE MATH SCIENCE FEMALE

READ 1.0000 0.6286 0.6793 0.6907 -0.0417
WRITE 0.6286 1.0000 0.6327 0.5691 0.2443
MATH 0.6793 0.6327 1.0000 0.6495 -0.0482

SCIENCE 0.6907 0.5691 0.6495 1.0000 -0.1382
FEMALE -0.0417 0.2443 -0.0482 -0.1382 1.0000

Correlations Between the Psychological Measurements and the Academic Measurements

READ WRITE MATH

LOCUS_OF_CONTROL 0.3736 0.3589 0.3373
SELF_CONCEPT 0.0607 0.0194 0.0536
MOTIVATION 0.2106 0.2542 0.1950

Correlations Between the Psychological Measurements and the Academic Measurements

SCIENCE FEMALE

LOCUS_OF_CONTROL 0.3246 0.1134
SELF_CONCEPT 0.0698 -0.1260
MOTIVATION 0.1157 0.0981

Canonical Correlation Analysis

Adjusted Approximate Squared
Canonical Canonical Standard Canonical

Correlation Correlation Error Correlation

1 0.464086 0.455474 0.032059 0.215376
2 0.167509 . 0.039712 0.028059
3 0.103991 . 0.040417 0.010814

Eigenvalues of Inv(E)*H
= CanRsq/(1-CanRsq)

Eigenvalue Difference Proportion Cumulative

1 0.2745 0.2456 0.8734 0.8734
2 0.0289 0.0179 0.0919 0.9652
3 0.0109 0.0348 1.0000

Test of H0: The canonical correlations in the current row and all that follow are zero

Likelihood Approximate
Ratio F Value Num DF Den DF Pr > F

1 0.75436113 11.72 15 1634.7 <.0001
2 0.96142996 2.94 8 1186 0.0029
3 0.98918584 2.16 3 594 0.0911

Multivariate Statistics and F Approximations

S=3 M=0.5 N=295

Statistic Value F Value Num DF Den DF Pr > F

Wilks' Lambda 0.75436113 11.72 15 1634.7 <.0001
Pillai's Trace 0.25424936 11.00 15 1782 <.0001
Hotelling-Lawley Trace 0.31429738 12.38 15 1113 <.0001
Roy's Greatest Root 0.27449563 32.61 5 594 <.0001

NOTE: F Statistic for Roy's Greatest Root is an upper bound.

Canonical Correlation Analysis

Raw Canonical Coefficients for the Psychological Measurements

Psych1 Psych2 Psych3

LOCUS_OF_CONTROL locus of control 1.2538339076 0.6214775237 -0.661689607
SELF_CONCEPT self-concept -0.35134993 1.1876866562 0.8267209411
MOTIVATION motivation 1.2624203286 -2.027264053

2.0002284379

Raw Canonical Coefficients for the Academic Measurements

Academic1 Academic2 Academic3

READ reading score 0.0446205959 0.0049100176 0.0213805581
WRITE writing score 0.0358771125 -0.042071471 0.0913073288
MATH math score 0.0234171847 -0.004229472 0.0093982096
SCIENCE science score 0.0050251567 0.0851621751 -0.109835018
FEMALE 0.6321192387 -1.084642482 -1.794646917

Canonical Correlation Analysis

Standardized Canonical Coefficients for the Psychological Measurements

Psych1 Psych2 Psych3

LOCUS_OF_CONTROL locus of control 0.8404 0.4166 -0.4435
SELF_CONCEPT self-concept -0.2479 0.8379 0.5833
MOTIVATION motivation 0.4327 -0.6948 0.6855

Standardized Canonical Coefficients for the Academic Measurements

Academic1 Academic2 Academic3

READ reading score 0.4508 0.0496 0.2160
WRITE writing score 0.3490 -0.4092 0.8881
MATH math score 0.2205 -0.0398 0.0885
SCIENCE science score 0.0488 0.8266 -1.0661
FEMALE 0.3150 -0.5406 -0.8944

Canonical Structure

Correlations Between the Psychological Measurements and Their Canonical Variables

Psych1 Psych2 Psych3

LOCUS_OF_CONTROL locus of control 0.9040 0.3897

-0.1756

SELF_CONCEPT self-concept 0.0208 0.7087 0.7052

MOTIVATION motivation 0.5672 -0.3509 0.7451

Correlations Between the Academic Measurements and Their Canonical Variables

Academic1 Academic2 Academic3

READ reading score 0.8404 0.3588 0.1354

WRITE writing score 0.8765 -0.0648 0.2546

MATH math score 0.7639 0.2979 0.1478

SCIENCE science score 0.6584 0.6768 -0.2304

FEMALE 0.3641 -0.7549 -0.5434

Correlations Between the Psychological Measurements and
the Canonical Variables of the Academic Measurements

Academic1 Academic2 Academic3

LOCUS_OF_CONTROL locus of control 0.4196 0.0653 -0.0183

SELF_CONCEPT self-concept 0.0097 0.1187 0.0733

MOTIVATION motivation 0.2632 -0.0588 0.0775

Correlations Between the Academic Measurements and the
Canonical Variables of the Psychological Measurements

Psych1 Psych2 Psych3

READ reading score 0.3900 0.0601 0.0141
WRITE writing score 0.4068 -0.0109 0.0265
MATH math score 0.3545 0.0499 0.0154
SCIENCE science score 0.3056 0.1134 -0.0240
FEMALE 0.1690 -0.1265 -0.0565

Canonical Redundancy Analysis

Raw Variance of the Psychological Measurements Explained by
Their Own The Opposite
Canonical Variables Canonical Variables
Canonical
Variable Cumulative Canonical Cumulative
Number Proportion Proportion R-Square Proportion Proportion

1 0.3806 0.3806 0.2154 0.0820 0.0820

2 0.3126 0.6932 0.0281 0.0088 0.0908

3 0.3068 1.0000 0.0108 0.0033 0.0941

Raw Variance of the Academic Measurements Explained by

Their Own The Opposite

Canonical Variables Canonical Variables

Canonical

Variable Cumulative Canonical Cumulative

Number Proportion Proportion R-Square Proportion Proportion

1 0.6251 0.6251 0.2154 0.1346 0.1346

2 0.1704 0.7955 0.0281 0.0048 0.1394

3 0.0395 0.8350 0.0108 0.0004 0.1398

Canonical Redundancy Analysis

Standardized Variance of the Psychological Measurements Explained by

Their Own The Opposite

Canonical Variables Canonical Variables

Canonical
Variable Cumulative Canonical Cumulative
Number Proportion Proportion R-Square Proportion Proportion

1 0.3798 0.3798 0.2154 0.0818 0.0818
2 0.2591 0.6389 0.0281 0.0073 0.0891
3 0.3611 1.0000 0.0108 0.0039 0.0930

Standardized Variance of the Academic Measurements Explained by
Their Own The Opposite
Canonical Variables Canonical Variables
Canonical
Variable Cumulative Canonical Cumulative
Number Proportion Proportion R-Square Proportion Proportion

1 0.5249 0.5249 0.2154 0.1130 0.1130
2 0.2499 0.7748 0.0281 0.0070 0.1201
3 0.0907 0.8655 0.0108 0.0010 0.1210

Canonical Redundancy Analysis

Squared Multiple Correlations Between the Psychological Measurements and the First M Canonical Variables of the Academic Measurements

M 1 2 3

LOCUS_OF_CONTROL locus of control 0.1760 0.1803 0.1806
SELF_CONCEPT self-concept 0.0001 0.0142 0.0196
MOTIVATION motivation 0.0693 0.0727 0.0787

Squared Multiple Correlations Between the Academic Measurements and the First M Canonical Variables of the Psychological Measurements

M 1 2 3

READ reading score 0.1521 0.1557 0.1559
WRITE writing score 0.1655 0.1656 0.1663
MATH math score 0.1257 0.1282 0.1284
SCIENCE science score 0.0934 0.1062 0.1068
FEMALE 0.0286 0.0445 0.0477

Correlations
Among the Original Variables

The SAS System

The CANCORR Procedure

Correlations Among the Original Variables

Correlations Among the Psychological Measurementsa

LOCUS_OF_
CONTROL SELF_CONCEPT MOTIVATION

LOCUS_OF_CONTROL 1.0000 0.1712 0.2451
SELF_CONCEPT 0.1712 1.0000 0.2886
MOTIVATION 0.2451 0.2886 1.0000

Correlations Among the Academic Measurementsb

READ WRITE MATH SCIENCE FEMALE

READ 1.0000 0.6286 0.6793 0.6907 -0.0417
WRITE 0.6286 1.0000 0.6327 0.5691 0.2443
MATH 0.6793 0.6327 1.0000 0.6495 -0.0482
SCIENCE 0.6907 0.5691 0.6495 1.0000 -0.1382
FEMALE -0.0417 0.2443 -0.0482 -0.1382 1.0000

Correlations Between the Psychological Measurements and the Academic Measurementsc

READ WRITE MATH

LOCUS_OF_CONTROL 0.3736 0.3589 0.3373
SELF_CONCEPT 0.0607 0.0194 0.0536
MOTIVATION 0.2106 0.2542 0.1950

Correlations Between the Psychological Measurements and the Academic Measurements

SCIENCE FEMALE

LOCUS_OF_CONTROL 0.3246 0.1134
SELF_CONCEPT 0.0698 -0.1260
MOTIVATION 0.1157 0.0981

a. Correlations Among the Psychological Measurements - This is the
Pearson correlation matrix for the three psychological variables. This
gives us a sense of the relationships between the variables within this group.
Because there are three variables in this group, the

correlation matrix is 3×3.
The psychological variables are not highly correlated. This suggests that
knowing the values in one of the psychological variables does not provide much
information about the other psychological variables. These relationships
between the variables will effect the way in which the group is summarized as a
linear combination of these variables.

b. Correlations Among the Academic Measurements - This is the Pearson
correlation matrix for the four academic variables and female. This
gives us a sense of the relationships between the variables within this group. Because there are three variables in this group, the correlation matrix is 5×5. We can see that the four standardized test variables (read, write, math,
and science) are much more highly correlated than the psychological
variables.

c. Correlations Between the Psychological Measurements and the Academic Measurements

- This matrix presents the psychological variables in rows and the academic variables in columns. The correlations in the matrix are between all combinations of variables in different groups. Because we have 3 variables in one group and 5 in the other, a total of 15 such correlations exist. In this table, we can see that all of the correlations are less than 0.4.

Canonical Correlations

	Adjusted Approximate Canonical Correlationd	Canonical Correlatione	Standard Errorf	Canonical Correlationg
1	0.464086	0.455474	0.032059	0.215376
2	0.167509	.	0.039712	0.028059
3	0.103991	.	0.040417	0.010814

Eigenvalues of Inv(E)*H

= CanRsq/(1-CanRsq)

Eigenvalueh Differencei Proportionj Cumulativek

1 0.2745 0.2456 0.8734 0.8734
2 0.0289 0.0179 0.0919 0.9652
3 0.0109 0.0348 1.0000

Test of H0: The canonical correlations in the current row and all that follow are zero

Likelihood Approximate
Ratiol F Valuem Num DF Den DFn Pr > Fo

1 0.75436113 11.72 15 1634.7 <.0001
2 0.96142996 2.94 8 1186 0.0029
3 0.98918584 2.16 3 594 0.0911

Multivariate Statistics and F Approximations

S=3 M=0.5 N=295

Statistic Value F Valuem Num DF Den DFn Pr > Fo

Wilks' Lambdap 0.75436113 11.72 15 1634.7 <.0001
Pillai's Traceq 0.25424936 11.00 15 1782 <.0001
Hotelling-Lawley Tracer 0.31429738 12.38 15 1113

<.0001

Roy's Greatest Roots 0.27449563 32.61 5 594 <.0001

NOTE: F Statistic for Roy's Greatest Root is an upper bound.

d.

Canonical Correlation -

These are the Pearson correlations of the pairs of canonical variates. The first pair of variates, a linear combination of the psychological measurements and a linear combination of the academic measurements, has a correlation coefficient of 0.464086. The second pair has a correlation coefficient of 0.167509, and the third pair 0.103991.

e.

Adjusted Canonical Correlation -

These are adjusted canonical correlations which are less biased than the raw correlations. These adjusted values may be negative. If an

adjusted canonical correlation is close to zero or if it is greater than the
previous adjusted canonical correlation, then it is reported as missing.

f.
Approximate Standard Error -
These are the approximate standard errors for the canonical correlations.

g.
Squared Canonical Correlation -
These are the squares of the canonical correlations. For example,
(0.464086*0.464086) = 0.215376.
These values can be interpreted similarly to R-squared values in OLS regression:
they are the proportion of the variance in the canonical variate of one set of
variables explained by the canonical variate of the other set of variables.

h.
Eigenvalue -
These are the eigenvalues of the product of the model

matrix and the inverse of
the error matrix. These eigenvalues can also be calculated using the squared
canonical correlations. The largest eigenvalue is equal to largest squared
correlation /(1- largest squared correlation). So 0.215376/(1-0.215376) =
0.2745. These calculations can be completed for each correlation to find
the corresponding eigenvalue. The magnitudes of the eigenvalues are related to
the tests of the correlations. The larger the eigenvalues are associated
with lower p-values. If we think about the relationship between the canonical
correlations and the eigenvalues, it makes sense that the larger correlations
are more likely to be significantly different from zero.

i.
Difference -
This is the difference between the given eigenvalue and the next-largest
eigenvalue: 0.2745-0.0289 = 0.2456 and 0.0289-0.0109 =

0.0179 (with rounding).

j.

Proportion -

This is the proportion of the sum of the eigenvalues represented by a given

eigenvalue. The sum of the three eigenvalues is (0.2745+0.0289+0.0109) =

0.3143. Then, the proportions can be calculated: 0.2745/0.3143 = 0.8734,

0.0289/0.3143 = 0.0919, and 0.0109/0.3143 = 0.0348.

k.

Cumulative -

This is the cumulative sum of the proportions.

l.

Likelihood Ratio -

This is the likelihood ratio for testing the hypothesis that the given canonical

correlation and all smaller ones are equal to zero in the population. It is

equivalent to Wilks' lambda (see superscript p) and can be calculated as the product of the values of

(1-canonical correlation2). In this example, our

canonical
correlations are 0.4641, 0.1675, and 0.1040. Hence the likelihood ratio for testing
that all three of the correlations are zero is (1-0.46412)*(1-0.16752)*(1-0.10402)
= 0.754361. To test that the two smaller canonical correlations, 0.1675
and 0.1040, are zero in the population, the likelihood is (1-0.16752)*(1-0.10402)
= 0.96143. The likelihood that the smallest canonical correlation is zero is (1-0.10402) = 0.989186.

m.
(Approximate) F Value -
These are the F values associated with the various tests (likelihood ratio or
one of the four multivariate tests) that are included in SAS's cancorr
procedure. For the likelihood ratio tests, the F values are approximate.
For Roy's Greatest Root, the F value is an upper bound.
For the likelihood tests, the F values are testing the hypotheses that the given canonical correlation and all smaller ones are equal

to zero in the population. For the multivariate tests, the F values are
testing the hypothesis that all three canonical correlations are equal to zero
in the population.

n.
Num DF, Den DF -
These are the degrees of freedom used in determining the F values. Note
that there are instances in which the degrees of freedom may be a
non-integer (here, the Den DF associated with Wilks' lambda is a
non-integer) because these degrees of freedom are calculated using the mean
squared errors, which are often non-integers.

o.
Pr > F
-
This is the p-value associated with the F value of a given test statistic. The null hypothesis that our two sets of variables are not linearly related is

**evaluated with regard to this p-value. The null hypothesis is rejected if
the p-value is less than our specified alpha level (often 0.05). If not, then we fail to
reject the null hypothesis. In this example, we reject the null hypothesis that all three canonical
correlations are equal to zero at alpha level 0.05 because the p-values for all tests of this hypothesis
are less than 0.05 (Wilks' Lambda,
Pillai's Trace, Hotelling-Lawley Trace,
Roy's Greatest Root and the
first Likelihood Ratio). The p-value associated with the likelihood ratio test of the second and third
canonical correlations suggest that they we can also reject the hypothesis that
both the second and third canonical correlations are zero, but the p-value
associated with the likelihood ratio test of the third canonical correlation
alone is 0.0911. Because this is greater than 0.05, we fail to reject the
hypothesis that the third canonical correlation is zero.**

p.

**Wilks' Lambda - This is one of the four multivariate statistics
calculated by SAS to test the null hypothesis that the canonical correlations
are zero (which, in turn, means that there is no linear relationship between the
two specified groups of variables). Wilks' lambda is the product of the values of
(1-canonical correlation2). In this example, our canonical
correlations are 0.4641, 0.1675, and 0.1040 so the Wilks' Lambda testing
all three of the correlations is (1-0.46412)*(1-0.16752)*(1-0.10402)
= 0.75436113. This test statistic is equal to the likelihood ratio (see
superscript l).**

q.

**Pillai's Trace - Pillai's trace is another of the four multivariate
statistics calculated by SAS. Pillai's trace is the sum of the squared canonical
correlations: 0.46412 + 0.16752 + 0.10402 =**

0.25424936.

r.

Hotelling-Lawley Trace - This is very similar to Pillai's trace. It is the sum of the values of (canonical correlation2/(1-canonical correlation2)). We can calculate 0.46412 /(1- 0.46412) + 0.16752/(1-0.16752) + 0.10402/(1-0.10402) = 0.31429738.

s.

Roy's Greatest Root - This is the largest eigenvalue. Because it is based on a maximum, it can behave differently from the other three test statistics. In instances where the other three are not significant and Roy's is significant, the effect should be considered not significant.

Canonical Coefficients

Raw Canonical Coefficients for the Psychological Measurementst

Psych1 Psych2 Psych3

LOCUS_OF_CONTROL locus of control 1.2538339076 0.6214775237 -0.661689607

SELF_CONCEPT self-concept -0.35134993 1.1876866562 0.8267209411

MOTIVATION motivation 1.2624203286 -2.027264053 2.0002284379

Raw Canonical Coefficients for the Academic Measurementst

Academic1 Academic2 Academic3

READ reading score 0.0446205959 0.0049100176 0.0213805581

WRITE writing score 0.0358771125 -0.042071471 0.0913073288

MATH math score 0.0234171847 -0.004229472 0.0093982096

SCIENCE science score 0.0050251567 0.0851621751 -0.109835018

FEMALE 0.6321192387 -1.084642482 -1.794646917

Standardized Canonical Coefficients for the Psychological Measurementsu

Psych1 Psych2 Psych3

LOCUS_OF_CONTROL locus of control 0.8404 0.4166

-0.4435

SELF_CONCEPT self-concept -0.2479 0.8379 0.5833

MOTIVATION motivation 0.4327 -0.6948 0.6855

Standardized Canonical Coefficients for the Academic Measurementsu

Academic1 Academic2 Academic3

READ reading score 0.4508 0.0496 0.2160

WRITE writing score 0.3490 -0.4092 0.8881

MATH math score 0.2205 -0.0398 0.0885

SCIENCE science score 0.0488 0.8266 -1.0661

FEMALE 0.3150 -0.5406 -0.8944

t.

Raw Canonical Coefficients for the Psychological/Academic Measurements -

These are the raw canonical coefficients. They define the linear relationship

between the variables in a given group and the canonical variates. They can be interpreted in the same way you would interpret regression coefficients,

assuming the canonical variate as the outcome variable. For example, a one

unit increase in locus_of_control leads to a 1.253834

**unit increase in
the first variate of the psychological measurements ("Psych1"), and a one unit
increase in read
score leads to a 0.0446206 unit increase in the first variate of the academic
measurements ("Academic1").**

**u.
Standardized Canonical Coefficients for the Psychological/Academic Measurements
-
These are the standardized canonical coefficients. This means that, if all of
the variables in the analysis are rescaled to have a mean of zero and a standard
deviation of 1, the coefficients generating the canonical variates would
indicate how a one standard deviation increase in the variable would change the
variate. For example, an increase of one standard deviation in
locus_of_control
would lead to a 0.8404 unit increase in the first variate**

**of the psychological
measurements ("Psych1"), and an increase of one standard deviation in
read
would lead to a 0.4508 unit increase in the first variate of the academic
measurements ("Academic1").**

Correlations Among Original Variables and Canonical Variates

**Correlations Between the Psychological Measurements and Their Canonical Variablesv
Psych1 Psych2 Psych3
LOCUS_OF_CONTROL locus of control 0.9040 0.3897 -0.1756
SELF_CONCEPT self-concept 0.0208 0.7087 0.7052
MOTIVATION motivation 0.5672 -0.3509 0.7451**

**Correlations Between the Academic Measurements and Their Canonical Variablesv
Academic1 Academic2 Academic3
READ reading score 0.8404 0.3588 0.1354
WRITE writing score 0.8765 -0.0648 0.2546
MATH math score 0.7639 0.2979 0.1478
SCIENCE science score 0.6584 0.6768 -0.2304**

FEMALE 0.3641 -0.7549 -0.5434

Correlations Between the Psychological Measurements and
the Canonical Variables of the Academic Measurementsw
Academic1 Academic2 Academic3
LOCUS_OF_CONTROL locus of control 0.4196 0.0653 -0.0183
SELF_CONCEPT self-concept 0.0097 0.1187 0.0733
MOTIVATION motivation 0.2632 -0.0588 0.0775

Correlations Between the Academic Measurements and the
Canonical Variables of the Psychological Measurementsx
Psych1 Psych2 Psych3
READ reading score 0.3900 0.0601 0.0141
WRITE writing score 0.4068 -0.0109 0.0265
MATH math score 0.3545 0.0499 0.0154
SCIENCE science score 0.3056 0.1134 -0.0240
FEMALE 0.1690 -0.1265 -0.0565

v. Correlations Between the Psychological/Academic

Measurements and Their Canonical Variables

- Here, SAS presents the correlations between each variable in a group and the group's canonical variates. These can allow us to see if the variates are combining the variables in such a way that might represent a particular idea. For example, we can see that the first variate for the psychological variables, Psych1,is highly correlated with locus_of_control and motivation, but uncorrelated with self-concept. Thus, this variate arguably captures much of the shared variance of locus_of_control and motivation. If we look at the academic variables, we can see that the first variate is highly correlated with all four of the subject variables. Those four variables were very highly correlated with each other (see superscript b), so it is not surprising that they should all be highly correlated with a variate that captures their

shared variance. The second variate is highly correlated with science and
negatively correlated with female. Thus, the first variate might
represent overall academic performance with an emphasis on reading and writing,
while the second variate emphasizes performance in science and is possibly
indicative of male students.

w.
Correlations Between the Psychological Measurements and the Canonical Variables of the Academic Measurements
- In addition to the correlations between the variables in a group and the
group's canonical variates, SAS also presents the correlations between each
variable in one group and the canonical variates of the other. We see that
the psychological variables locus_of_control,
self_concept and
motivation
are correlated with Academic1, Academic2 and

**Academic3 (a total of 3×3=9
correlations). Here, we can see that locus_of_control and
motivation are correlated with the first academic variate, while
self_concept is uncorrelated with the first variate but slightly correlated
with the second variate. Based on our observations about these two variates in
superscript v, we might interpret these correlations to mean that overall
academic performance, especially reading and writing, are related to
locus_of_control and motivation, while performance in science and
gender may be related to self_concept.**

**x.
Correlations Between the Academic Measurements and the Canonical Variables of the Psychological Measurements**

**- Here, we see how the academic variables
read,
write,**

math,
science
and female
are correlated with Psych1, Psych2 and Psych3 (a total of 5×3=15 correlations).
We see that
the academic variables read, write, math and science
are all correlated with Psych1, the first psychological variate strongly
correlated with locus_of_control and motivation. This supports
what we noted in superscript w about the possible relationship between
overall academic performance and these two psychological variables.

Canonical Redundancy Analysis

Canonical Redundancy Analysis

Raw Variance of the Psychological Measurements Explained by
Their Own The Opposite
Canonical Variablesy Canonical Variablesz
Canonical

Variable Cumulative Canonical Cumulative
Number Proportion Proportion R-Square Proportion Proportion
1 0.3806 0.3806 0.2154 0.0820 0.0820
2 0.3126 0.6932 0.0281 0.0088 0.0908
3 0.3068 1.0000 0.0108 0.0033 0.0941

Raw Variance of the Academic Measurements Explained by
Their Own The Opposite
Canonical Variablesy Canonical Variablesz
Canonical
Variable Cumulative Canonical Cumulative
Number Proportion Proportion R-Square Proportion Proportion
1 0.6251 0.6251 0.2154 0.1346 0.1346
2 0.1704 0.7955 0.0281 0.0048 0.1394
3 0.0395 0.8350 0.0108 0.0004 0.1398

Standardized Variance of the Psychological Measurements Explained by
Their Own The Opposite
Canonical Variablesaa Canonical Variablesbb
Canonical

Variable Cumulative Canonical Cumulative
Number Proportion Proportion R-Square Proportion Proportion
1 0.3798 0.3798 0.2154 0.0818 0.0818
2 0.2591 0.6389 0.0281 0.0073 0.0891
3 0.3611 1.0000 0.0108 0.0039 0.0930

Standardized Variance of the Academic Measurements Explained by
Their Own The Opposite
Canonical Variablesaa Canonical Variablesbb
Canonical
Variable Cumulative Canonical Cumulative
Number Proportion Proportion R-Square Proportion Proportion
1 0.5249 0.5249 0.2154 0.1130 0.1130
2 0.2499 0.7748 0.0281 0.0070 0.1201
3 0.0907 0.8655 0.0108 0.0010 0.1210

y.
Raw Variance of the Psychological/Academic Measurements Explained by
Their Own Canonical Variables -
This is the degree to which the canonical variates of a

group can explain the
variability in the group's variables. For example, we see here that the
first canonical variate for the academic group explains 62.5% of the variability
in the academic variables and the first canonical variate for the
psychological group explains 38% of the variability in the psychological variables.

z.
Raw Variance of the Psychological/Academic Measurements Explained by
The Opposite Canonical Variables -
This is the degree to which the canonical variates of a group can explain the
variability in the other
group's variables. For example, we see here that the first canonical
variate for the academic group explains 8.2% of the variability in the
psychological variables and the first canonical variate for the psychological
group explains 13.5% of the variability in the academic

variables.

aa.
Standardized Variance of the Psychological/Academic Measurements Explained by
Their Own Canonical Variables -
This is similar to superscript y, but performed on standardized data variables.

bb.
Standardized Variance of the Psychological/Academic Measurements Explained by
The Opposite Canonical Variables -This
is similar to superscript z, but performed on standardized data variables.

Squared Multiple Correlations

Squared Multiple Correlations Between the Psychological Measurements
and the First M Canonical Variables of the Academic Measurementscc
M 1 2 3
LOCUS_OF_CONTROL locus of control 0.1760 0.1803 0.1806

SELF_CONCEPT self-concept 0.0001 0.0142 0.0196
MOTIVATION motivation 0.0693 0.0727 0.0787

Squared Multiple Correlations Between the Academic Measurements and
the First M Canonical Variables of the Psychological Measurementscc
M 1 2 3
READ reading score 0.1521 0.1557 0.1559
WRITE writing score 0.1655 0.1656 0.1663
MATH math score 0.1257 0.1282 0.1284
SCIENCE science score 0.0934 0.1062 0.1068
FEMALE 0.0286 0.0445 0.047

cc.
Squared Multiple Correlations Between the Psychological/Academic Measurements and
the First M Canonical Variables of the Psychological Measurements -
Here, the correlations that were presented earlier between each variable in a
given group and the canonical variates of the other group, are squared.
Each value is equivalent to the R-squared value in an

**OLS regression where we
are predicting a single variable with a single variate or vice versa. For
example, we saw earlier in the output that
locus_of_control
and Academic1 have a correlation of 0.4196. We can calculate
(0.4196*0.4196) = 0.1760, the squared correlation presented in this portion of
the output. This means that 17.6% of the variability in locus_of_control
can be explained by Academic1.**

**For more on the options available in
cancorr
and details on the underlying calculations, see the corresponding
SAS documentation page.**