

What is Pillai's Trace?

Authored by
stats writer

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What is Pillai's Trace? (Definition & Example)

A is used to determine whether or not different levels of an explanatory variable lead to statistically different results in some .

For example, we might be interested in understanding whether or not three levels of education (Associate's degree, Bachelor's degree, Master's degree) lead to annual incomes. In this case, we have one explanatory variable and one response variable.

A is an extension of the one-way ANOVA in which there is more than one response variable. For example, we might be interested in understanding whether education leads to different annual incomes *and* different amounts of student loan debt. In this case, we have one explanatory variable and two response variables.

One of the test statistics that is produced by a MANOVA is Pillai's trace.

What is Pillai's Trace?

Pillai's trace is a test statistic produced by a MANOVA. It is a value that ranges from 0 to 1.

The closer Pillai's trace is to 1, the stronger the evidence that the explanatory variable has a statistically significant effect on the values of the response variables.

Pillai's trace, often denoted V , is calculated as:

$$V = \text{trace}(H(H+E)^{-1})$$

where:

H : The hypothesis sum of squares and cross products matrix

E : The error sum of squares and cross products matrix

When performing a MANOVA, most statistical software will use Pillai's trace to calculate a rough approximation to an F-statistic along with a corresponding p-value.

If this p-value is less than some significance level (i.e. $\alpha = .05$) then we reject the null hypothesis of the MANOVA and conclude that the explanatory variable has a significant effect on the values of the response variables.

When to Use Pillai's Trace

When performing a MANOVA, most statistical software will actually produce four test statistics:

Pillai's Trace

Wilks' Lambda

Lawley-Hotelling Trace

Roy's Largest Root

It is recommended to use Pillai's trace as the test statistic when the assumptions of a MANOVA are violated. As a reminder, MANOVA makes the following assumptions:

The follow a multivariate normal probability distribution with means equal to zero.

The variance-covariance matrices of each group of residuals are equal.

The are independent.

When one or more of these assumptions are violated, Pillai's trace tends to be the most robust test statistic.

Example of Calculating Pillai's Trace

In , we perform a MANOVA in Stata using the following

variables:

Explanatory variable: Level of education (Associate, Bachelor, or Master)

Response variables: Annual income, Total student loan debt

The following screenshot shows the output of the MANOVA:

```
. manova income debt = educ
```

Number of obs = 24

W = Wilks' lambda

L = Lawley-Hotelling trace

P = Pillai's trace

R = Roy's largest root

Source	Statistic	df	F(df1,	df2) =	F	Prob>F
educ	W 0.4433	2	4.0	40.0	5.02	0.0023 e
	P 0.5588		4.0	42.0	4.07	0.0071 a
	L 1.2510		4.0	38.0	5.94	0.0008 a
	R 1.2472		2.0	21.0	13.10	0.0002 u
Residual		21				
Total		23				

e = exact, a = approximate, u = upper bound on F

Note that the MANOVA produced four test statistics:

Wilks' lambda: F-Statistic = 5.02, P-value = 0.0023.

Pillai's trace: F-Statistic = 4.07, P-value = 0.0071.

Lawley-Hotelling trace: F-Statistic = 5.94, P-value =

0.0008.

Roy's largest root: F-Statistic = 13.10, P-value = 0.0002.

The F-value for each test statistic varies but each corresponding p-value is less than .05, so we would reject the null hypothesis of the MANOVA and conclude that level of education has a significant effect on annual income and total student loan debt.

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