What is Negative Binomial Regression and how is it used in R for data analysis?

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Negative Binomial Regression is a statistical method used for analyzing count data, where the outcome variable is a non-negative integer (such as number of deaths, number of accidents, etc.). It is a type of generalized linear model that allows for overdispersion, meaning the variance of the data is greater than the mean.

In R, the negative binomial regression model can be fitted using the "glm.nb" function from the "MASS" package. This function allows for the specification of a negative binomial distribution and can handle both continuous and categorical predictors. The resulting model provides estimates of the effects of each predictor on the outcome variable, as well as goodness-of-fit measures.

Negative binomial regression is commonly used in research and data analysis for a variety of applications, such as predicting the number of customer complaints in a business, or the number of infections in a population. It is particularly useful when the data has a high number of zeros and a large range of counts, where traditional regression methods may fail. It is also a popular choice for analyzing longitudinal count data, where repeated measurements are taken over time. In summary, negative binomial regression is a powerful tool for analyzing count data in R and allows for accurate and efficient modeling of data with overdispersion.

Negative Binomial Regression | R Data Analysis Examples

Negative binomial regression is for modeling count variables, usually for over-dispersed count outcome variables.

This page uses the following packages. Make sure that you can load them before trying to run the examples on this page. If you do not have a package installed, run: `install.packages("packagename")`, or if you see the version is out of date, run: `update.packages()`.

```
require(foreign)
require(ggplot2)
require(MASS)
```

```
Version info: Code for this page was tested in R Under development (unstable)
(2013-01-06 r61571)
On: 2013-01-22
With: MASS 7.3-22; ggplot2 0.9.3; foreign 0.8-52; knitr 1.0.5
```

Please note: The purpose of this page is to show how to use various data analysis commands. It does not cover all aspects of the research process which researchers are expected to do. In particular, it does not cover data cleaning and checking, verification of assumptions, model diagnostics or potential follow-up analyses.

Examples of negative binomial regression

Example 1. School administrators study the attendance behavior of high school juniors at two schools. Predictors of the number of days of absence include the type of program in which the student is enrolled and a standardized

test in math.

Example 2. A health-related researcher is studying the number of hospital visits in past 12 months by senior citizens in a community based on the characteristics of the individuals and the types of health plans under which each one is covered.

Description of the data

Let's pursue Example 1 from above.

We have attendance data on 314 high school juniors from two urban high schools in the file `nb_data`. The response variable of interest is days absent, `daysabs`. The variable `math` gives the standardized math score for each student. The variable `prog` is a three-level nominal variable indicating the type of instructional program in which the student is enrolled.

Let's look at the data. It is always a good idea to start with descriptive statistics and plots.

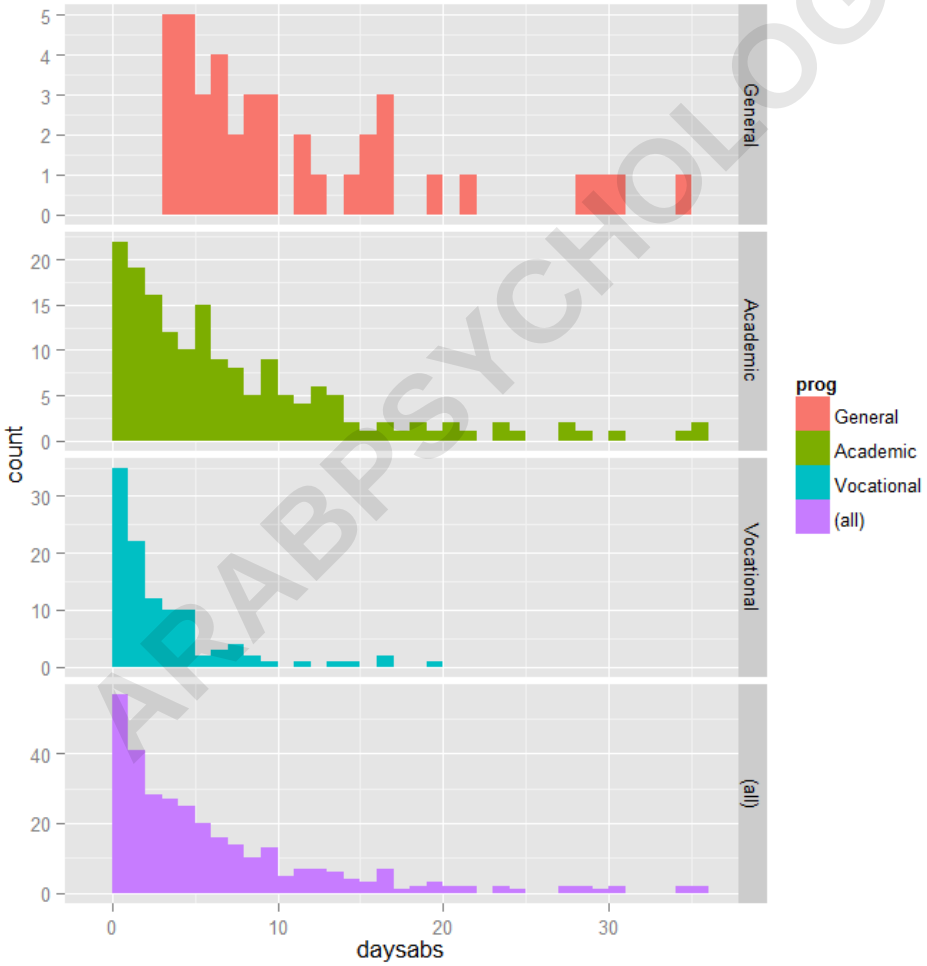
```
dat <- read.dta("https://stats.idre.ucla.edu/stat/stata/dae/nb_data.dta")
dat <- within(dat, {
prog <- factor(prog, levels = 1:3, labels = c("General", "Academic", "Vocational"))
id <- factor(id)
})

summary(dat)


## id gender math daysabs
## 1001 : 1 female:160 Min. : 1.0 Min. : 0.00
## 1002 : 1 male :154 1st Qu.:28.0 1st Qu.: 1.00
## 1003 : 1 Median :48.0 Median : 4.00
## 1004 : 1 Mean :48.3 Mean : 5.96
## 1005 : 1 3rd Qu.:70.0 3rd Qu.: 8.00
## 1006 : 1 Max. :99.0 Max. :35.00
## (Other):308
## prog
## General : 40
## Academic :167
## Vocational:107
##
```

##

##

##

```
ggplot(dat, aes(daysabs, fill = prog)) +
geom_histogram(binwidth = 1) + facet_grid(prog ~
., margins = TRUE, scales = "free")
```



Each variable has 314 valid observations and their

distributions seem quite reasonable.
The unconditional mean of our outcome variable is much lower than its variance.

Let's continue with our description of the variables in this dataset.
The table below shows the average numbers of days absent by program type
and seems to suggest that program type is a good candidate for predicting the number of
days absent, our outcome variable, because the mean value of the outcome appears to vary by
`prog`. The variances within each level of `prog` are
higher than the means within each level. These are the conditional means and
variances. These differences suggest that over-dispersion is present and that a
Negative Binomial model would be appropriate.

```
with(dat, tapply(daysabs, prog, function(x) {
sprintf("M (SD) = %1.2f (%1.2f)", mean(x), sd(x))
}))
```

```
## General Academic Vocational
```

"M (SD) = 10.65 (8.20)" "M (SD) = 6.93 (7.45)" "M (SD) = 2.67 (3.73)"

Analysis methods you might consider

Below is a list of some analysis methods you may have encountered. Some of the methods listed are quite reasonable, while others have
either fallen out of favor or have limitations.

Negative binomial regression analysis

Below we use the `glm.nb` function from the `MASS` package to estimate a negative binomial regression.

```
summary(m1 <- glm.nb(daysabs ~ math + prog, data = dat))


##
## Call:
## glm.nb(formula = daysabs ~ math + prog, data = dat,
init.theta = 1.032713156,
## link = log)
##
## Deviance Residuals:
## Min 1Q Median 3Q Max
```

```
## -2.155 -1.019 -0.369 0.229 2.527
##
## Coefficients:
## Estimate Std. Error z value Pr(>|z|)
## (Intercept) 2.61527 0.19746 13.24 < 2e-16 ***
## math -0.00599 0.00251 -2.39 0.017 *
## progAcademic -0.44076 0.18261 -2.41 0.016 *
## progVocational -1.27865 0.20072 -6.37 1.9e-10 ***
## ---
## Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## (Dispersion parameter for Negative Binomial(1.033) family taken to be 1)
##
## Null deviance: 427.54 on 313 degrees of freedom
## Residual deviance: 358.52 on 310 degrees of freedom
## AIC: 1741
##
## Number of Fisher Scoring iterations: 1
##
##
## Theta: 1.033
## Std. Err.: 0.106
```

```
##
## 2 x log-likelihood: -1731.258

m2 <- update(m1, . ~ . - prog)
anova(m1, m2)

## Likelihood ratio tests of Negative Binomial Models
##
## Response: daysabs
## Model theta Resid. df 2 x log-lik. Test df LR stat.
## 1 math 0.8559 312 -1776
## 2 math + prog 1.0327 310 -1731 1 vs 2 2 45.05
## Pr(Chi)
## 1
## 2 1.652e-10
```

Checking model assumption

As we mentioned earlier, negative binomial models assume the conditional means
are not equal to the conditional variances. This inequality is captured by
estimating a dispersion parameter (not shown in the output) that is held

constant in a Poisson model. Thus, the Poisson model is actually nested in the negative binomial model. We can then use a likelihood ratio test to compare these two and test this model assumption. To do this, we will run our model as a Poisson.

```
m3 <- glm(daysabs ~ math + prog, family = "poisson", data = dat)
pchisq(2 * (logLik(m1) - logLik(m3)), df = 1, lower.tail = FALSE)
```

```
## 'log Lik.' 2.157e-203 (df=5)
```

In this example the associated chi-squared value estimated from 2*(logLik(m1) - logLik(m3)) is 926.03 with one degree of freedom. This strongly suggests the negative binomial model, estimating the dispersion parameter, is more appropriate than the Poisson model.

We can get the confidence intervals for the coefficients by

profiling the likelihood function.

(est <- cbind(Estimate = coef(m1), confint(m1)))

Waiting for profiling to be done...

Estimate 2.5 % 97.5 %
(Intercept) 2.615265 2.2421 3.012936
math -0.005993 -0.0109 -0.001067
progAcademic -0.440760 -0.8101 -0.092643
progVocational -1.278651 -1.6835 -0.890078

We might be interested in looking at incident rate ratios rather than
coefficients. To do this, we can exponentiate our model coefficients. The same
applies to the confidence intervals.

exp(est)

Estimate 2.5 % 97.5 %
(Intercept) 13.6708 9.4127 20.3470
math 0.9940 0.9892 0.9989
progAcademic 0.6435 0.4448 0.9115

progVocational 0.2784 0.1857 0.4106

The output above indicates that the incident rate for `prog` = 2
is 0.64 times the incident rate for the reference group (`prog` = 1).
Likewise, the incident rate for `prog` = 3 is 0.28 times the incident
rate for the reference group holding the other variables constant. The
percent change in the incident rate of `daysabs` is a 1% decrease
for every unit increase in `math`.

The form of the model equation for negative binomial regression is
the same as that for Poisson regression. The log of the expected outcome is
predicted with a linear combination of the predictors:

where $I(prog_i = j)$ is an indicator function such that if $prog_i = j$ is equal to 1 and otherwise is equal to 0, for $j in {2, 3}$. Therefore,

The coefficients have an *additive* effect in the $ln(y)$ scale and the IRR have a *multiplicative* effect in the y scale. The dispersion parameter in negative binomial regression does not affect the expected counts, but it does affect the estimated variance of the expected counts. More details can be found in the Modern Applied Statistics with S by W.N. Venables and B.D. Ripley (the book companion of the `MASS` package).

For additional information on the various metrics in which the results can be presented, and the interpretation of such, please see Regression Models for Categorical Dependent Variables Using Stata, Second Edition by J. Scott Long and Jeremy Freese (2006).

Predicted values

For assistance in further understanding the model, we

can look at predicted
counts for various levels of our predictors. Below we create new datasets with
values of `math` and `prog` and then use the `predict` command to calculate the predicted number of events.

First, we can look at predicted counts for each value of `prog` while
holding `math` at its mean.
To do this, we create a new dataset with the combinations of `prog` and
`math` for which we would like to find predicted values, then use the `predict`
command.

```
newdata1 <- data.frame(math = mean(dat$math), prog =
factor(1:3, levels = 1:3,
labels = levels(dat$prog)))
newdata1$phat <- predict(m1, newdata1, type =
"response")
newdata1
```

```
## math prog phat
## 1 48.27 General 10.237
```

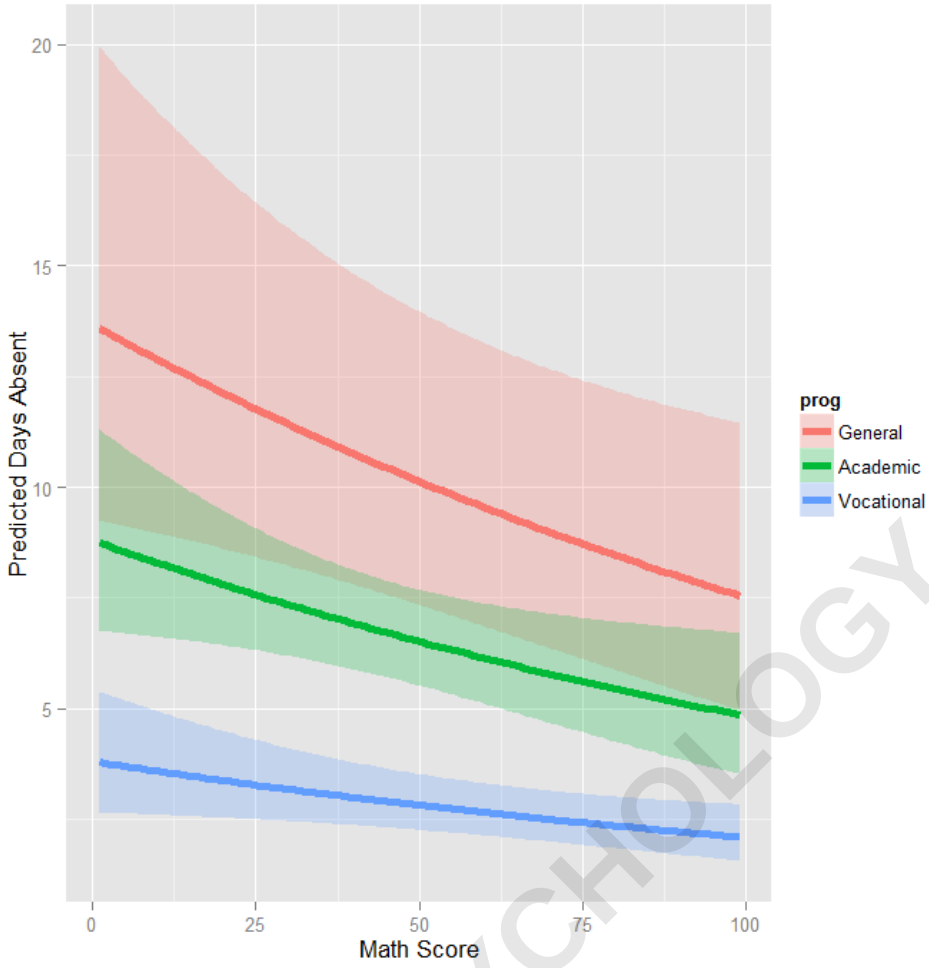
2 48.27 Academic 6.588
3 48.27 Vocational 2.850

In the output above, we see that the predicted number of events (e.g., days
absent) for a general program is about 10.24, holding `math` at its mean. The predicted
number of events for an academic program is lower at 6.59, and the
predicted number of events for a vocational program is about 2.85.

Below we will obtain the mean predicted number of events for values of `math`
across its entire range for each level of `prog` and graph these.

```
newdata2 <- data.frame(
math = rep(seq(from = min(dat$math), to = max(dat$math), length.out = 100), 3),
prog = factor(rep(1:3, each = 100), levels = 1:3, labels = levels(dat$prog)))

newdata2 <- cbind(newdata2, predict(m1, newdata2, type = "link", se.fit=TRUE))
```

```
newdata2 <- within(newdata2, {
DaysAbsent <- exp(fit)
LL <- exp(fit - 1.96 * se.fit)
UL <- exp(fit + 1.96 * se.fit)
})

ggplot(newdata2, aes(math, DaysAbsent)) +
geom_ribbon(aes(ymin = LL, ymax = UL, fill = prog),
alpha = .25) +
geom_line(aes(colour = prog), size = 2) +
labs(x = "Math Score", y = "Predicted Days Absent")
```



The graph shows the expected count across the range of math scores,
for each type of program along with 95 percent confidence intervals.
Note that the lines are not straight because this is a log linear model, and
what is plotted are the expected values, not the log of the expected values.

Things to consider

References

See also