What is a Uniform Distribution?

Authored by

stats writer

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The **Uniform Distribution** is one of the simplest and most fundamental types of probability distributions in statistics. It is defined by the principle that all potential outcomes within a specified range are equally likely to occur. This contrasts sharply with distributions like the Normal Distribution, where outcomes cluster around a central mean.

The Uniform Distribution is sometimes referred to as the **Rectangular Distribution** because of the shape of its probability density function (PDF) plot. When graphed, the PDF forms a rectangle, indicating a constant probability density across its entire span. This makes it an ideal model for situations where there is absolutely no prior information or bias suggesting that one outcome is more probable than another.

Common examples of processes that follow a Uniform Distribution include basic random events such as rolling a fair die (where results 1 through 6 are equally likely), flipping a coin, and drawing a card from a deck of cards. In mathematical notation, a random variable X following a continuous uniform distribution is often denoted as U(a, b), where a is the minimum possible value and b is the maximum possible value.

Fundamentally, the **uniform distribution** describes a scenario where every value between a defined interval from a to b is equally likely to be observed. This constant likelihood across the interval is the defining characteristic of this distribution type.

Understanding the Uniform Probability Formula

When dealing with a continuous uniform distribution, calculating the probability of an outcome falling within a specific sub-interval is straightforward. Since the probability density is constant across the entire range (from a to b), the probability of observing a value within a certain window (from $x1$ to $x2$) is simply the ratio of the length of that window to the length of the entire distribution interval.

If a random variable X adheres to a continuous uniform distribution, the probability that X falls between two values, $x1$ and $x2$ (where $a \leq x1 < x2 \leq b$), is determined by the following fundamental formula:

P(x1 < X < x2) = (x2 - x1) / (b - a)

This formula leverages the geometric interpretation of the distribution. The probability corresponds to the area of the rectangle formed by the desired interval (numerator) relative to the total area of the distribution (denominator).

The variables used in the formula are defined as follows:

x1: Represents the lower boundary of the specific range of interest.

x2: Represents the upper boundary of the specific range of interest.
a: Denotes the minimum possible value of the entire distribution (the lower bound).
b: Denotes the maximum possible value of the entire distribution (the upper bound).

Case Study: Calculating Probability of Dolphin Weight

To illustrate the practical application of the uniform probability formula, consider a scenario involving marine biology. Suppose that the weight of a certain population of dolphins is known to be uniformly distributed between 100 pounds (the minimum value, *a*) and 150 pounds (the maximum value, *b*). This assumption implies that a dolphin weighing 101 pounds is just as likely as one weighing 149 pounds.

If a researcher randomly selects a dolphin from this population, we can calculate the probability that the chosen animal's weight falls within a specific range, for instance, between 120 and 130 pounds. Here, *x1* = 120 and *x2* = 130.

Applying the formula, we substitute the known variables into the equation. The range of interest (130 - 120) is 10 pounds, and the total range of the distribution (150 - 100) is 50 pounds. The calculation proceeds as follows:

$P(120 < X < 130) = (130 - 120) / (150 - 100)$
$P(120 < X < 130) = 10 / 50$
$P(120 < X < 130) = \mathbf{0.2}$

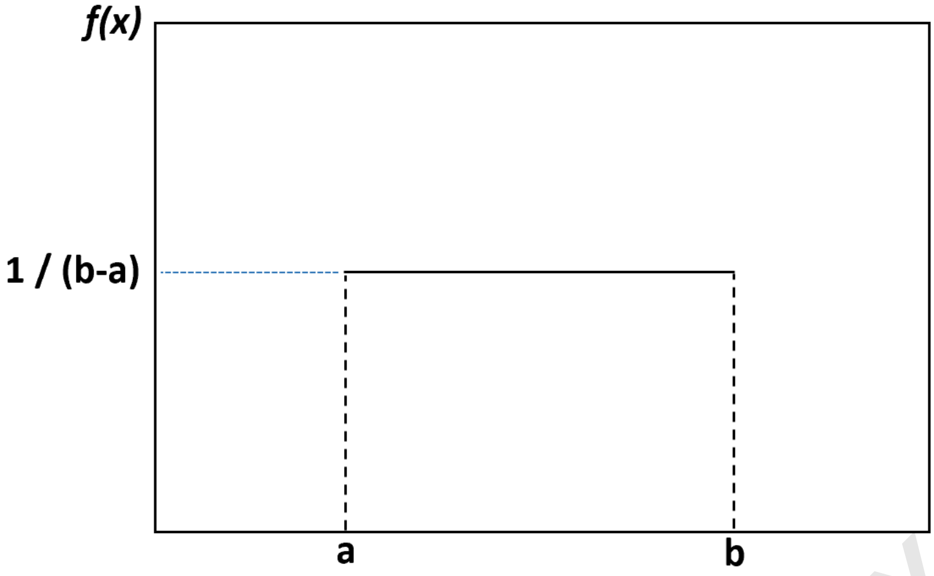
The resulting probability is 0.2, or 20%. This means there is a one-in-five chance that a randomly selected dolphin will weigh between 120 and 130 pounds, given the uniform nature of the weight distribution.

Visualizing the Uniform Distribution

The shape of a probability density function provides crucial insight into how outcomes are distributed. For the uniform distribution, the visualization is distinctively rectangular, which gives rise to its alternative name, the Rectangular Distribution. This density plot shows a constant height (density) across the entire range from the lower bound (*a*) to the upper bound (*b*).

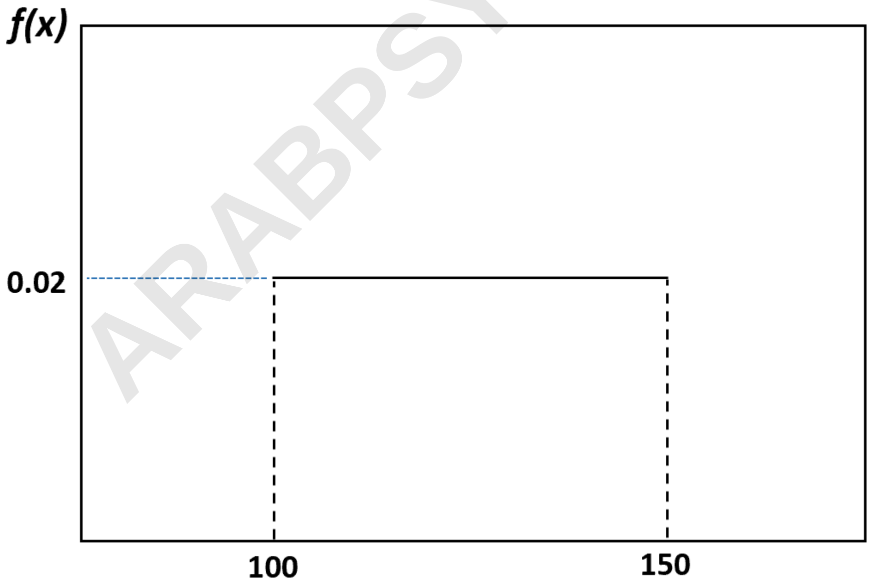
When we plot the density function, the height of the rectangle is equal to 1 / (b - a). This ensures that the total area under the curve is exactly 1, a requirement for any valid probability distribution. Outside of the defined interval , the probability density is precisely zero, meaning it is impossible for the random variable *X* to take on a value beyond these limits.

If we create a density plot to visualize the uniform distribution across a general interval , it would visually represent a flat, constant density as shown below:



Returning to our previous example, where dolphin weights are uniformly distributed between 100 pounds (a) and 150 pounds (b), we can visualize the specific parameters of this distribution. The density remains constant across the 50-pound range.

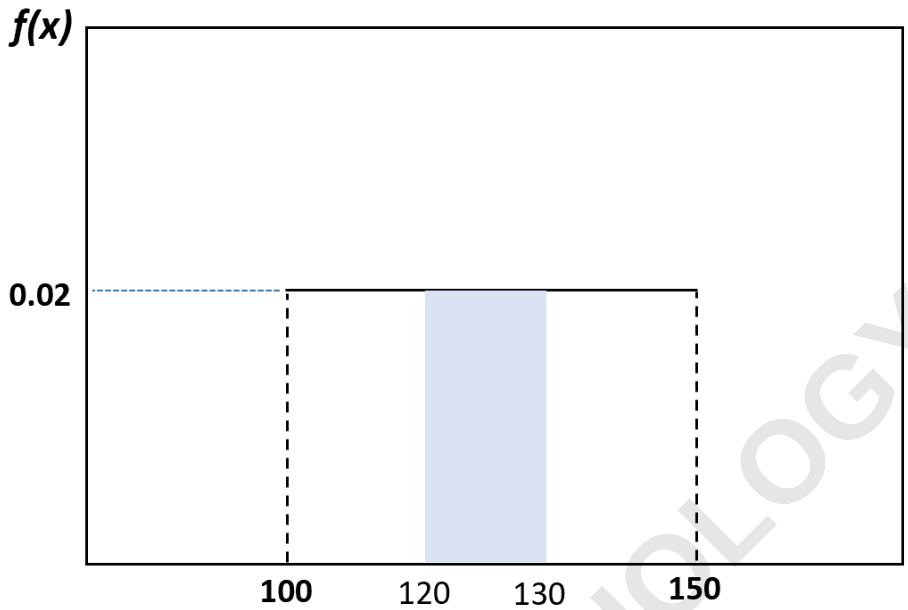
Here is the visualization demonstrating the distribution of dolphin weights between 100 and 150 pounds:



Furthermore, probability calculations in a uniform distribution correspond directly to the area under the curve. When we calculated the probability of a dolphin weighing between 120 and 130 pounds

(P = 0.2), this probability is represented by the shaded area over that specific interval on the density plot.

The calculated probability (0.2) for the interval can be visualized as the shaded region in the following plot:



Statistical Properties of the Uniform Distribution

Beyond calculating probabilities for specific intervals, it is essential to understand the core statistical measures that characterize the uniform distribution. These properties--including the mean, median, standard deviation, and variance--are crucial for comparing the uniform model to other probability models and understanding its spread.

Because the density is constant, the distribution is perfectly symmetrical around its center point. Consequently, the mean (expected value) and the median (the 50th percentile) must coincide exactly at the midpoint of the interval .

The key properties of a continuous uniform distribution defined over the interval are derived using integral calculus and are summarized below:

Mean (Expected Value): This is the average value of the distribution, calculated simply as the midpoint: **(a + b) / 2**
Median: Due to symmetry, the median is identical to the mean: **(a + b) / 2**
Variance: This measures the spread of the data points relative to the mean: **(b - a)2 / 12**
Standard Deviation: The square root of the Variance, representing the average distance of values

from the mean: **√(b - a)2 / 12**

Calculating Descriptive Statistics

Using the same dolphin weight example--where weights are uniformly distributed between a minimum of *a* = 100 pounds and a maximum of *b* = 150 pounds--we can now calculate the descriptive statistics for this specific population using the formulas above.

These calculations provide crucial insights into the central tendency and the variability of the dolphin weights. Knowing the Standard Deviation and the Variance allows us to quantify the expected spread of weights around the average.

Here is the step-by-step calculation of the key properties for the dolphin weight distribution (U(100, 150)):

Mean weight: (a + b) / 2 = (150 + 100) / 2 = **125** pounds. This is the expected average weight.
Median weight: (a + b) / 2 = (150 + 100) / 2 = **125** pounds. Half the dolphins weigh more than 125 pounds, and half weigh less.
Variance of weight: (150 - 100)2 / 12 = 502 / 12 = 2500 / 12 = **208.33** (in pounds squared).
Standard Deviation of weight: √(150 - 100)2 / 12 = √208.33 ≈ **14.43** pounds. This figure represents the typical variation of weights from the mean.

Uniform Distribution Practice Problems and Solutions

To solidify your understanding of the continuous uniform distribution, work through the following practice problems. Remember to always define your interval endpoints (*a* and *b*) and your specific range of interest (*x1* and *x2*) before applying the core probability formula: P(x1 < X < x2) = (x2 - x1) / (b - a).

These scenarios demonstrate how the uniform model is applicable to real-world situations involving time, distance, and weight, assuming constant probability density over the given range.

Question 1: A bus shows up at a bus stop every 20 minutes. If you arrive at the bus stop randomly, what is the probability that the bus will show up in 8 minutes or less?

Solution 1: Since the bus arrival time is continuous and random within the 20-minute cycle, this follows a uniform distribution. The minimum waiting time (*a*) is 0 minutes, and the maximum waiting time (*b*) is 20 minutes. We are interested in the range between 0 (*x1*) and 8 (*x2*) minutes.

Thus, we calculate the probability as:

P(0 < X < 8) = (8-0) / (20-0) = 8/20 = **0.4**.

Question 2: The length of an NBA game, excluding overtime, is uniformly distributed between 120 minutes and 170 minutes. What is the probability that a randomly selected NBA game lasts more than 155 minutes?

Solution 2: The total range is defined by *a* = 120 minutes and *b* = 170 minutes. Since we want the game to last "more than 155 minutes," our specific interval runs from *x1* = 155 up to the maximum possible time, *x2* = 170 minutes.

Thus, we calculate the probability as:

$P(155 < X < 170) = (170-155) / (170-120) = 15/50 =$ **0.3**.

Question 3: The weight of a certain species of frog is uniformly distributed between 15 grams and 25 grams. If you randomly select a frog, what is the probability that the frog weighs between 17 and 19 grams?

Solution 3: Here, the distribution is defined by *a* = 15 grams and *b* = 25 grams. The specific range of interest is defined by *x1* = 17 grams and *x2* = 19 grams.

Thus, we calculate the probability as:

$P(17 < X < 19) = (19-17) / (25-15) = 2/10 =$ **0.2**.

Note: Specialized calculators and statistical software can be used to check your answers quickly for each of these uniform distribution problems.