

What is a Probability Mass Function (PMF) in Statistics?

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A Probability Mass Function (PMF) is a statistical tool used to describe the probability distribution of a discrete random variable. It assigns a probability to each possible outcome of the variable, giving a complete picture of the likelihood of each outcome occurring. This function is often represented graphically as a histogram or bar chart, with the height of each bar representing the probability of that specific outcome. PMFs are commonly used in data analysis to make predictions and determine the likelihood of certain events or outcomes. They are an essential tool in understanding and analyzing discrete data in statistics.

What is a Probability Mass Function (PMF) in Statistics?

A probability mass function, often abbreviated PMF, tells us the probability that a takes on a certain value.

For example, suppose we roll a dice one time. If we let x denote the number that the dice lands on, then the probability that the x is equal to different values can be described as follows:

$P(X=1): 1/6$ $P(X=2): 1/6$ $P(X=3): 1/6$ $P(X=4): 1/6$ $P(X=5): 1/6$ $P(X=6): 1/6$

There is an equal chance that the dice could land on any number between 1 and 6.

Here's how we would write these probabilities as a probability mass function:

$$p_X(x) = \begin{cases} 1/6 & \text{if } x = 1 \\ 1/6 & \text{if } x = 2 \\ 1/6 & \text{if } x = 3 \\ 1/6 & \text{if } x = 4 \\ 1/6 & \text{if } x = 5 \\ 1/6 & \text{if } x = 6 \\ 0 & \text{otherwise} \end{cases}$$

The left side of the diagram shows the probability associated with the outcomes on the right side:

	Probabilities	Outcomes
$p_X(x) = \left\{ \begin{array}{l} \end{array} \right.$	1/6	if $x = 1$
	1/6	if $x = 2$
	1/6	if $x = 3$
	1/6	if $x = 4$
	1/6	if $x = 5$
	1/6	if $x = 6$
	0	otherwise

One characteristic of a probability mass function is that all of the probabilities must add up to 1. You'll notice that this PMF satisfies that condition:

$$\text{Sum of probabilities} = 1/6 + 1/6 + 1/6 + 1/6 + 1/6 + 1/6 = 1.$$

The support of a probability mass function refers to the set of values that the discrete random variable can take.

In this example, the support would be {1, 2, 3, 4, 5, 6} since the value of the dice can take on any of these values.

Outside of the support, the value for the PMF is equal to zero. For example, the probability that the dice lands on "0" or "7" or "8" is equal to zero since none of these numbers are included in the support.

Probability Mass Functions in Practice

The two most common examples of probability mass functions in practice are for the and the .

Binomial Distribution

If a random variable X follows a Binomial distribution, then the probability that $X = k$ successes can be found by the following formula:

$$P(X=k) = nCk * p^k * (1-p)^{n-k}$$

n : number of trials
 k : number of successes
 p : probability of success on a given trial
 nCk : the number of ways to obtain k successes in n trials

For example, suppose we flip a coin 3 times. We can

use the formula above to determine the probability of obtaining 0, 1, 2, and 3 heads during these 3 flips:

$$\begin{aligned} P(X=0) &= {}^3C_0 * .50 * (1-.5)^{3-0} = 1 * 1 * (.5)^3 = 0.125 \\ P(X=1) &= {}^3C_1 * .51 * (1-.5)^{3-1} = 1 * 1 * (.5)^2 = 0.375 \\ P(X=2) &= {}^3C_2 * .52 * (1-.5)^{3-2} = 1 * 1 * (.5)^1 = 0.375 \\ P(X=3) &= {}^3C_3 * .53 * (1-.5)^{3-3} = 1 * 1 * (.5)^0 = 0.125 \end{aligned}$$

Poisson Distribution

If a random variable X follows a Poisson distribution, then the probability that $X = k$ successes can be found by the following formula:

$$P(X=k) = \lambda^k * e^{-\lambda} / k!$$

where:

λ : mean number of successes that occur during a specific interval
 k : number of successes
 e : a constant equal to approximately 2.71828

For example, suppose a particular hospital experiences an average of 2 births per hour. We can use the formula above to determine the probability of experiencing 0, 1, 2, 3 births, etc. in a given hour:

$$\begin{aligned}P(X=0) &= 20 * e^{-2} / 0! = 0.1353 \\P(X=1) &= 21 * e^{-2} / 1! = 0.2707 \\P(X=2) &= 22 * e^{-2} / 2! = 0.2707 \\P(X=3) &= 23 * e^{-2} / 3! = 0.1805\end{aligned}$$

Visualizing a PMF

We often visualize probability mass functions with bar charts.

For example, the following bar chart shows the probabilities associated with the number of births per hour for the Poisson distribution described in the previous example: