

How to Easily Understand Two-Tailed Hypothesis Tests: 3 Example Problems

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Two-tailed hypothesis tests are a fundamental type of statistical test crucial for determining if a statistically significant difference exists between two conditions or groups. Unlike their directional counterparts, two-tailed tests are designed to detect differences in **both positive and negative directions**, meaning they assess deviation from a null value without specifying the direction of that change. This article will provide a detailed explanation of their underlying principles and walk through three practical example problems demonstrating how these tests are applied in real-world scenarios, such as evaluating new treatments or comparing population metrics.

Understanding the Foundation of Hypothesis Testing

In the realm of statistics, hypothesis testing is the formal procedure used to evaluate whether a specific claim or statement regarding a population parameter is supported by sample data. This structured methodology allows researchers to make evidence-based decisions about populations based on observed samples. Every robust hypothesis test necessitates the formal definition of two competing statements: the null hypothesis and the alternative hypothesis.

These two hypotheses define the scope of the investigation. The null hypothesis (H_0) represents the status quo--a statement of no effect or no difference--while the alternative hypothesis (H_a) challenges this status quo, suggesting that a significant effect or difference exists. These formal definitions guide the interpretation of the test results and are crucial for determining statistical significance.

Defining the Null and Alternative Hypotheses

When preparing any statistical investigation, the formulation of the competing hypotheses must adhere to strict logical frameworks. The hypotheses are structured around the targeted population parameter (often represented by μ , the population mean) and take on distinct forms depending on whether a one-tailed or two-tailed approach is required.

The core forms for these hypotheses are established as follows:

H_0 (Null Hypothesis): Population parameter = $\leq$, $\geq$ some value. This hypothesis always asserts a statement of equality or non-difference, representing the status quo being tested.

H_a (Alternative Hypothesis): Population parameter $<$, $>$, $\neq$ some value. This hypothesis represents the critical claim the researcher is attempting to validate using statistical evidence.

Differentiating One-Tailed and Two-Tailed Tests

The distinction between the two major types of hypothesis tests--one-tailed and two-tailed--rests entirely on the form of the alternative hypothesis (H_a). This choice determines whether we are

looking for an effect in one direction only or if we are interested in any statistically meaningful change.

One-tailed test: This approach is directional. The alternative hypothesis uses either the "less than" ($<$) or "greater than" ($>$) sign, specifying the expected direction of the effect.

Two-tailed test: This approach is non-directional. The alternative hypothesis always contains the "not equal" ($\neq$) sign, seeking to confirm if the parameter is **different** from the hypothesized value, regardless of whether it is higher or lower.

The defining feature of a **two-tailed test** is that the alternative hypothesis includes the "not equal" ($\neq$) sign. This signifies that the researcher is testing for any divergence from the null hypothesis, whether that effect is positive or negative. Consequently, the rejection region for the null hypothesis is distributed across both extremes of the sampling distribution, making the test sensitive to discrepancies on either side.

To solidify this concept and see how statistical results are interpreted in a non-directional context, we will now examine three comprehensive examples.

Example 1: Assessing Factory Widget Weights

Consider a scenario where the standard operating procedure at a factory is assumed to yield a specific widget with an average weight of exactly **20 grams**. A production engineer introduces a new, optimized manufacturing method and wants to know if this change has affected the mean weight. Crucially, the engineer is unsure whether the weight will increase or decrease; they only hypothesize that the new method causes the average weight to be **different** from 20 grams.

Since the inquiry is non-directional, a two-tailed hypothesis test is appropriate. The hypotheses are formally stated as follows, where μ represents the true mean weight of the widgets produced by the new method:

H₀ (Null Hypothesis): The average weight remains 20 grams ($\mu = 20$ grams).

H_A (Alternative Hypothesis): The average weight is different from 20 grams ($\mu \neq 20$ grams).

This setup confirms it is a **two-tailed hypothesis test** because the alternative hypothesis contains the "not equal" ($\neq$) sign. The engineer proceeds by using the new method to produce a sample of 20 widgets and collects the necessary descriptive statistics for analysis using a t-test.

The collected sample data is:

Sample size (n): **20** widgets

Sample mean weight ($\bar{x}$): **19.8** grams

Sample standard deviation (s): **3.1** grams

Upon plugging these values into the test formula, the resulting calculated values are:

t-test statistic: **-0.288525**

Two-tailed p-value: **0.776**

We compare the p-value (0.776) to the standard significance level ($\alpha = 0.05$). Since 0.776 is significantly larger than 0.05, the engineer **fails to reject the null hypothesis**. This means the data does not provide sufficient statistical evidence to conclude that the true mean weight of the widgets produced by the new method is statistically different from the assumed standard of 20 grams.

Example 2: Testing Novel Fertilizer Effects on Plant Growth

A previous study established that a specific plant species achieves an average growth of **10 inches** when treated with the standard fertilizer. A botanist has developed a new, experimental fertilizer and wishes to determine if it significantly alters the growth rate. The botanist's primary hypothesis is that the new fertilizer will lead to an average growth that is **not equal** to 10 inches, but she has no strong theoretical basis to predict whether the growth will be higher or lower.

This lack of directional prediction mandates the use of a two-tailed test. The established average growth rate (10 inches) serves as the null value, against which the experimental results will be compared. The formal hypotheses are constructed as follows:

H₀ (Null Hypothesis): The average growth using the new fertilizer is 10 inches ($\mu = 10$ inches).

H_A (Alternative Hypothesis): The average growth using the new fertilizer is different from 10 inches ($\mu \neq 10$ inches).

To test this claim, she applies the new fertilizer to a simple random sample of 15 plants and measures their growth. This scenario perfectly illustrates a **two-tailed hypothesis test** because the critical question is whether the new treatment deviates in either direction from the existing mean.

The data collected from the sample is summarized below:

Sample size (n): **15** plants

Sample mean growth ($\bar{x}$): **11.4** inches

Sample standard deviation (s): **2.5** inches

By applying the appropriate statistical formula, the resulting calculated values for the test are determined:

t-test statistic: **2.1689**

Two-tailed p-value: **0.0478**

In this case, the calculated p-value (0.0478) is compared against the standard alpha level ($\alpha = 0.05$). Since 0.0478 is smaller than 0.05, the botanist **rejects the null hypothesis**. This statistical decision indicates that she has sufficient evidence to conclude that the new fertilizer causes an average growth rate that is significantly different from the previously established 10 inches.

Example 3: Evaluating the Impact of a New Studying Technique

A university professor is investigating a novel studying technique. She knows that the historical mean score on a specific standardized exam is **82**, but she hypothesizes that the new technique will **influence** this mean score. Because she cannot definitively predict whether the technique will improve scores or hinder them, she must structure her investigation to detect differences in either direction.

The professor decides to implement a two-tailed hypothesis test to evaluate the impact. The hypotheses are set up based on the current population mean (82) and her expectation that the new method causes a change:

H₀ (Null Hypothesis): The mean exam score remains 82 ($\mu = 82$).

H_A (Alternative Hypothesis): The mean exam score is different from 82 ($\mu \neq 82$).

To gather empirical data, the professor enrolls 25 students into a cohort and instructs them to use the studying technique for one month leading up to the examination. This setup constitutes a **two-tailed approach** because the alternative hypothesis explicitly states that the true mean exam score under the new condition is not equal to the historical mean.

The gathered statistics from the sample of 25 students are as follows:

Sample size (n): **25**

Sample mean score (x): **85**

Sample standard deviation (s): **4.1**

After running the data through the statistical model, the resulting test values are calculated:

t-test statistic: **3.6586**

Two-tailed p-value: **0.0012**

Comparing the calculated p-value (0.0012) against the significance level ($\alpha = 0.05$), we observe that the p-value is considerably smaller than the threshold. Consequently, the professor decisively **rejects the null hypothesis**. She has collected compelling statistical evidence indicating that the new studying method produces exam scores with an average that is significantly different from the historical mean of 82.

Conclusion and Resources

The three examples above highlight the practical application of **two-tailed hypothesis tests** across various disciplines, including manufacturing quality control, agricultural science, and educational assessment. In each case, the core utility of the two-tailed approach was its ability to detect significant deviation from a population mean without requiring a prior directional assumption. This non-directional flexibility makes it a powerful tool for initial exploratory research or when assessing the overall impact of an intervention.

Understanding the precise formulation of the null (H_0) and alternative (H_a) hypotheses is essential for correctly applying and interpreting these tests. Remember that a rejection of H_0 in a two-tailed test simply means the parameter is statistically different from the hypothesized value--further investigation may still be required to determine the direction and practical significance of that difference.

For readers interested in deepening their knowledge of statistical methodology, the following resources provide additional information and tutorials regarding hypothesis testing fundamentals: