

How to Calculate Probability from Mean and Standard Deviation: A Step-by-Step Guide

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The Foundation: Understanding the Normal Distribution

Calculating the probability of a specific outcome requires a robust understanding of the underlying data distribution. In many real-world scenarios--such as measuring human height, test scores, or physical characteristics--the data tends to follow a specific pattern known as the Normal Distribution, often visualized as a symmetrical, bell-shaped curve. This distribution is fundamental to inferential statistics because it allows us to standardize and compare data points across different datasets, provided we know the distribution's central tendency and spread. Before diving into the calculation process, it is essential to confirm that the variable X in question is indeed normally distributed. This assumption is critical, as the methods described below, particularly the use of the z-score and the standard normal table, rely entirely on the properties inherent to the standard normal curve. If the data is skewed or heavily kurtotic, these methods may yield inaccurate results.

The beauty of the standard normal distribution lies in its predictability. If a variable is normally distributed, approximately 68% of its values fall within one standard deviation of the mean, 95% within two standard deviations, and 99.7% within three standard deviations. This principle, known as the Empirical Rule, provides a quick, rough estimate of probability. However, for precise calculations required in statistical analysis, we must employ a more rigorous method involving the standardization of the data point, which transforms any normal distribution into the standard normal distribution (with a mean of zero and a standard deviation of one). This transformation allows us to use universal lookup tables, making complex probability calculations accessible and consistent.

The Role of Mean and Standard Deviation in Probability

To calculate the exact probability of a random variable X falling within a certain range, we must first define the parameters of the distribution: the mean (μ) and the standard deviation (σ). The population mean (μ) represents the center or average of the dataset, indicating where the peak of the bell curve lies. It is the expected value of the random variable. The population standard deviation (σ), on the other hand, quantifies the amount of variation or dispersion of the dataset values. A smaller standard deviation indicates that the data points are tightly clustered around the mean, resulting in a narrow, tall bell curve, while a larger standard deviation signifies that the data points are spread out over a wider range, resulting in a flatter curve.

These two parameters are indispensable because they define the exact shape and position of the normal curve. Without them, it would be impossible to determine the relative likelihood of any single observation. The standard deviation, in particular, serves as the unit of measure when we normalize the data. By converting a raw data value (x) into a standardized score, we are effectively measuring its distance from the mean not in original units (e.g., kilograms or test points) but in units of standard deviations. This process of standardization is what enables us to move from an abstract, specific distribution (defined by μ and σ) to the universally applicable standard normal

distribution, where all calculations are performed using the z-score. This standardization is the critical link between raw data and verifiable probabilities.

Step-by-Step Guide: Calculating the Z-Score

The crucial first step in finding the probability associated with a specific value (x) in a normally distributed dataset is calculating the z-score. The z-score (also known as the standard score) quantifies how many standard deviations an individual data value falls above or below the mean. This single transformation simplifies complex probability problems immensely by reducing the infinite possibilities of normal distributions down to a single, standardized curve. A positive z-score means the value is above the mean, a negative z-score means it is below the mean, and a z-score of zero indicates the value is exactly the mean.

The formula for calculating the z-score is fundamental to this process. It isolates the deviation of the raw score from the mean and then scales that deviation by dividing it by the standard deviation. Mathematically, the calculation is represented as follows:

$$\mathbf{z\text{-score} = (x - \mu) / \sigma}$$

Where the variables represent:

- x:** The individual data value for which we seek the probability.
- μ :** The population mean (the center of the distribution).
- σ :** The population standard deviation (the measure of variability).

It is vital to perform the subtraction ($x - \mu$) first, which determines the raw distance from the mean. This difference is often referred to as the deviation score. Dividing this deviation score by σ converts the distance into standardized units of standard deviation. This standardization process effectively maps the raw score onto the standard normal distribution, preparing it for the next step: looking up the cumulative probability using the z-table. This is the only way to accurately determine the area under the curve to the left of our specific data point.

Interpreting and Using the Z-Table

Once the z-score has been accurately calculated, the second step involves referencing the standard normal distribution table, commonly referred to as the Z-table. This table provides the cumulative probability--that is, the area under the curve--to the left of the calculated z-score. This area represents the probability that a randomly selected value from the population will be less than the data value x . Understanding how to read and interpret the Z-table is paramount for solving probability problems involving normally distributed data, as it contains the precise probabilities derived from the mathematical function of the standard normal curve.

The Z-table typically lists z-scores in the rows and columns. The first decimal place of the z-score is usually found in the leftmost column, and the second decimal place is located in the top row. By finding the intersection of the appropriate row and column, you retrieve a value between 0 and 1. This value is the cumulative probability, $P(Z < z)$. For positive z-scores, you typically use a table that reflects probabilities greater than 0.5, and for negative z-scores, a table reflecting probabilities less than 0.5 is often used, although a single comprehensive table can handle both. When the problem asks for probability greater than x or probability between two values, simple algebraic manipulations (subtraction from 1, or subtraction between two probabilities) must be applied to the initial cumulative probability obtained from the table.

The following examples illustrate how to apply these two essential steps--calculating the z-score and using the Z-table--to solve common probability questions under different scenarios. Paying close attention to the wording of the question (e.g., "less than," "greater than," or "between") is crucial for determining the final calculation required after the table lookup.

Example 1: Calculating Probability Less Than X

Consider a scenario where the scores on a comprehensive standardized test are known to be normally distributed with a mean (μ) of 82 and a standard deviation (σ) of 8. We want to determine the probability that a randomly selected student scores less than 84 on this test. This is the simplest type of probability calculation because the result obtained directly from the Z-table represents the required probability.

Step 1: Determine the Z-Score.

First, we must standardize the score of 84 by calculating its corresponding z-score using the formula $(x - \mu) / \sigma$. This converts the raw score into a measure of standard deviation units:

$$z\text{-score} = (x - \mu) / \sigma = (84 - 82) / 8 = 2 / 8 = \mathbf{0.25}$$

Since the z-score is 0.25, the raw score of 84 is exactly 0.25 standard deviations above the mean score of 82.

Step 2: Use the Z-Table to Find the Cumulative Probability.

Next, we consult the standard normal distribution table for the value corresponding to a z-score of **0.25**. We locate 0.2 in the leftmost column and then move across to the column labeled 0.05. The intersecting value provides the cumulative probability $P(Z < 0.25)$.

z	0	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517
0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879
0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224
0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549
0.7	0.7580	0.7611	0.7642	0.7673	0.7704	0.7734	0.7764	0.7794	0.7823	0.7852
0.8	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106	0.8133
0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389
1.0	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621
1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8810	0.8830
1.2	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980	0.8997	0.9015
1.3	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162	0.9177
1.4	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9279	0.9292	0.9306	0.9319
1.5	0.9332	0.9345	0.9357	0.9370	0.9382	0.9394	0.9406	0.9418	0.9429	0.9441
1.6	0.9452	0.9463	0.9474	0.9484	0.9495	0.9505	0.9515	0.9525	0.9535	0.9545

Upon locating the value, we find that $P(Z < 0.25)$ is 0.5987. Since the question asks for the probability that a score is less than 84, this value directly represents the solution. Therefore, the probability that a given student scores less than 84 on the test is **0.5987**, or 59.87%.

Example 2: Calculating Probability Greater Than X

Imagine we are studying a certain species of penguin whose height is normally distributed with a mean (μ) of 30 inches and a standard deviation (σ) of 4 inches. We are tasked with finding the probability that a randomly selected penguin is greater than 28 inches tall. This scenario requires an extra step, as the Z-table only provides the cumulative probability (area to the left).

Step 1: Determine the Z-Score.

We first standardize the height of 28 inches to find the corresponding z-score:

$$z\text{-score} = (x - \mu) / \sigma = (28 - 30) / 4 = -2 / 4 = \mathbf{-0.5}$$

The negative sign confirms that 28 inches is half a standard deviation below the population mean of 30 inches.

Step 2: Use the Z-Table to Find the Corresponding Probability (Area to the Left).

Next, we look up the negative z-score of **-0.5** in the Z-table. This value gives us $P(Z < -0.5)$, which is the probability that a penguin is less than 28 inches tall:

z	0	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
-3.0	0.0013	0.0013	0.0013	0.0012	0.0012	0.0011	0.0011	0.0011	0.0010	0.0010
-2.9	0.0019	0.0018	0.0018	0.0017	0.0016	0.0016	0.0015	0.0015	0.0014	0.0014
-2.8	0.0026	0.0025	0.0024	0.0023	0.0023	0.0022	0.0021	0.0021	0.0020	0.0019
-2.7	0.0035	0.0034	0.0033	0.0032	0.0031	0.0030	0.0029	0.0028	0.0027	0.0026
-2.6	0.0047	0.0045	0.0044	0.0043	0.0041	0.0040	0.0039	0.0038	0.0037	0.0036
-2.5	0.0062	0.0060	0.0059	0.0057	0.0055	0.0054	0.0052	0.0051	0.0049	0.0048
-2.4	0.0082	0.0080	0.0078	0.0075	0.0073	0.0071	0.0069	0.0068	0.0066	0.0064
-2.3	0.0107	0.0104	0.0102	0.0099	0.0096	0.0094	0.0091	0.0089	0.0087	0.0084
-2.2	0.0139	0.0136	0.0132	0.0129	0.0125	0.0122	0.0119	0.0116	0.0113	0.0110
-2.1	0.0179	0.0174	0.0170	0.0166	0.0162	0.0158	0.0154	0.0150	0.0146	0.0143
-2.0	0.0228	0.0222	0.0217	0.0212	0.0207	0.0202	0.0197	0.0192	0.0188	0.0183
-1.9	0.0287	0.0281	0.0274	0.0268	0.0262	0.0256	0.0250	0.0244	0.0239	0.0233
-1.8	0.0359	0.0351	0.0344	0.0336	0.0329	0.0322	0.0314	0.0307	0.0301	0.0294
-1.7	0.0446	0.0436	0.0427	0.0418	0.0409	0.0401	0.0392	0.0384	0.0375	0.0367
-1.6	0.0548	0.0537	0.0526	0.0516	0.0505	0.0495	0.0485	0.0475	0.0465	0.0455
-1.5	0.0668	0.0655	0.0643	0.0630	0.0618	0.0606	0.0594	0.0582	0.0571	0.0559
-1.4	0.0808	0.0793	0.0778	0.0764	0.0749	0.0735	0.0721	0.0708	0.0694	0.0681
-1.3	0.0968	0.0951	0.0934	0.0918	0.0901	0.0885	0.0869	0.0853	0.0838	0.0823
-1.2	0.1151	0.1131	0.1112	0.1093	0.1075	0.1056	0.1038	0.1020	0.1003	0.0985
-1.1	0.1357	0.1335	0.1314	0.1292	0.1271	0.1251	0.1230	0.1210	0.1190	0.1170
-1.0	0.1587	0.1562	0.1539	0.1515	0.1492	0.1469	0.1446	0.1423	0.1401	0.1379
-0.9	0.1841	0.1814	0.1788	0.1762	0.1736	0.1711	0.1685	0.1660	0.1635	0.1611
-0.8	0.2119	0.2090	0.2061	0.2033	0.2005	0.1977	0.1949	0.1922	0.1894	0.1867
-0.7	0.2420	0.2389	0.2358	0.2327	0.2296	0.2266	0.2236	0.2206	0.2177	0.2148
-0.6	0.2743	0.2709	0.2676	0.2643	0.2611	0.2578	0.2546	0.2514	0.2483	0.2451
-0.5	0.3085	0.3050	0.3015	0.2981	0.2946	0.2912	0.2877	0.2843	0.2810	0.2776
-0.4	0.3446	0.3409	0.3372	0.3336	0.3300	0.3264	0.3228	0.3192	0.3156	0.3121
-0.3	0.3821	0.3783	0.3745	0.3707	0.3669	0.3632	0.3594	0.3557	0.3520	0.3483
-0.2	0.4207	0.4168	0.4129	0.4090	0.4052	0.4013	0.3974	0.3936	0.3897	0.3859

The cumulative probability corresponding to a z-score of -0.50 is **0.3085**. This figure represents the area under the curve to the left of 28 inches, meaning 30.85% of penguins are shorter than 28 inches.

Step 3: Calculate the Probability Greater Than X.

Since the problem asks for the probability that a penguin is **greater than** 28 inches tall, we need the area to the right of the z-score. Because the total area under the probability curve must equal 1 (100%), we subtract the cumulative probability (area to the left) from 1:

$$P(X > 28) = 1 - P(X < 28) = 1 - 0.3085 = \mathbf{0.6915}.$$

Therefore, the probability that a randomly selected penguin will have a height greater than 28 inches is 0.6915, or 69.15%.

Example 3: Calculating Probability Between Two Values

Let us consider the weight of a certain species of turtle, which is normally distributed with a mean (μ) of 400 pounds and a standard deviation (σ) of 25 pounds. We wish to find the probability that a randomly selected turtle weighs between 410 pounds and 425 pounds. Finding the area between two values requires calculating two separate z-scores and then finding the difference between their corresponding cumulative probabilities.

Step 1: Determine Both Z-Scores.

We must standardize both boundary values, 410 pounds ($x?$) and 425 pounds ($x?$):

Z-score for $x? = 410$: $(410 - 400) / 25 = 10 / 25 = \mathbf{0.4}$

Z-score for $x? = 425$: $(425 - 400) / 25 = 25 / 25 = \mathbf{1.0}$

This tells us that the desired weight range spans from 0.4 standard deviations above the mean to exactly 1 standard deviation above the mean.

Step 2: Use the Z-Table to Find Both Cumulative Probabilities.

We must find the cumulative probability for the upper value ($Z = 1.0$) and the cumulative probability for the lower value ($Z = 0.4$).

First, look up the z-score of **0.4**:

z	0	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517
0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879
0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224
0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549
0.7	0.7580	0.7611	0.7642	0.7673	0.7704	0.7734	0.7764	0.7794	0.7823	0.7852
0.8	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106	0.8133
0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389
1.0	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621
1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8810	0.8830
1.2	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980	0.8997	0.9015
1.3	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162	0.9177
1.4	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9279	0.9292	0.9306	0.9319
1.5	0.9332	0.9345	0.9357	0.9370	0.9382	0.9394	0.9406	0.9418	0.9429	0.9441
1.6	0.9452	0.9463	0.9474	0.9484	0.9495	0.9505	0.9515	0.9525	0.9535	0.9545
1.7	0.9554	0.9564	0.9573	0.9582	0.9591	0.9599	0.9608	0.9616	0.9625	0.9633
1.8	0.9641	0.9649	0.9656	0.9664	0.9671	0.9678	0.9686	0.9693	0.9699	0.9706

The cumulative probability $P(Z < 0.4)$ is **0.6554**. This is the probability of a turtle weighing less than 410 pounds.

Next, look up the z-score of **1.0**:

z	0	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517
0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879
0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224
0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549
0.7	0.7580	0.7611	0.7642	0.7673	0.7704	0.7734	0.7764	0.7794	0.7823	0.7852
0.8	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106	0.8133
0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389
1.0	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621
1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8810	0.8830
1.2	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980	0.8997	0.9015
1.3	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162	0.9177
1.4	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9279	0.9292	0.9306	0.9319
1.5	0.9332	0.9345	0.9357	0.9370	0.9382	0.9394	0.9406	0.9418	0.9429	0.9441
1.6	0.9452	0.9463	0.9474	0.9484	0.9495	0.9505	0.9515	0.9525	0.9535	0.9545
1.7	0.9554	0.9564	0.9573	0.9582	0.9591	0.9599	0.9608	0.9616	0.9625	0.9633
1.8	0.9641	0.9649	0.9656	0.9664	0.9671	0.9678	0.9686	0.9693	0.9699	0.9706

The cumulative probability $P(Z < 1.0)$ is **0.8413**. This is the probability of a turtle weighing less than 425 pounds.

Step 3: Calculate the Probability Between the Two Values.

To find the area between 410 and 425 pounds, we subtract the smaller cumulative probability ($P(Z < 0.4)$) from the larger cumulative probability ($P(Z < 1.0)$). This isolates the area under the curve corresponding to the middle range:

$$P(410 < X < 425) = P(Z < 1.0) - P(Z < 0.4) = 0.8413 - 0.6554 = \mathbf{0.1859}.$$

Thus, the probability that a randomly selected turtle weighs between 410 pounds and 425 pounds is 0.1859, or **18.59%**. This rigorous method ensures accuracy when analyzing ranges within any standard normal distribution.

Summary and Key Takeaways

The systematic approach of converting raw scores into z-scores and then referencing the Z-table is the cornerstone of probability calculations for normally distributed data. This method allows statisticians and analysts to utilize a single, universal table regardless of the original units of measurement or the specific mean and standard deviation of the population being studied. The key steps remain consistent across all problem types: first, standardize the data point(s) using the

defined mean and standard deviation; second, look up the cumulative probabilities; and third, perform the necessary algebraic subtraction or addition based on whether the question asks for the area to the left, to the right, or between two points.

Mastering the z-score formula and the nuances of the Z-table empowers practitioners to accurately model and predict outcomes in fields ranging from quality control and finance to biological sciences and psychology. It confirms the vital relationship between central tendency (mean) and variability (standard deviation) in defining the likelihood of any particular observation occurring within a statistically representative sample.

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