How to calculate the pooled variance?

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The calculation of **pooled variance** is a cornerstone technique in comparative statistics, specifically employed when analyzing data from two or more independent samples. It serves as a single, consolidated estimate of the common variance shared across these populations, based on the assumption that they exhibit **homogeneity of variance**. This method is fundamentally a form of weighted average, where the weights assigned to each sample's variance are proportional to their respective degrees of freedom, ensuring that larger or more reliable samples contribute more significantly to the final estimate.

To accurately determine the pooled variance, a systematic approach must be followed. Initially, the individual variance for each group must be calculated. Subsequently, each individual variance is weighted by multiplying it by its corresponding sample size minus one (its **degrees of freedom**). These weighted variances are then summed together, forming the numerator of the pooled variance formula. Finally, this sum is divided by the total degrees of freedom across all groups--the combined sample sizes minus the number of groups--to yield the robust pooled estimate. This process is essential for statistical tests that rely on a single, unbiased estimate of population dispersion.

Understanding the Concept of Pooled Variance

The core utility of **pooled variance** (s_p^2) lies in its ability to consolidate dispersion estimates from multiple samples into one representative value, provided that the underlying populations share the same true variance (σ^2). This practice is crucial in inferential statistics, especially when sample sizes are small or moderately unequal. By pooling the variances, we leverage the full scope of available data to obtain a more stable and accurate estimate of the population variability than any single sample could provide alone. This stability is directly linked to an increase in the associated degrees of freedom.

When conducting hypothesis testing, particularly comparisons between two group means, the standard error of the difference needs to be calculated. If we assume equal population variances, using the pooled variance allows us to use the same standard deviation estimate for both groups, simplifying the standard error calculation and leading to a more powerful statistical test. If the variances were significantly unequal, pooling would introduce bias, necessitating the use of alternative tests like the Welch's t-test, which does not assume equal variances.

The weighting mechanism inherent in the pooled calculation ensures statistical fairness. Since larger samples generally provide more precise estimates of the true population variance, the calculation weights each sample variance by its degrees of freedom ($n_i - 1$). This means that a sample of size $n=100$ will contribute significantly more to the final pooled estimate than a sample of size $n=10$. This weighted approach is what distinguishes the pooled variance from a simple arithmetic mean of the two sample variances.

Prerequisites: Homogeneity of Variance

The validity of using **pooled variance** rests entirely upon the critical assumption known as **homogeneity of variance** (or homoscedasticity). This assumption mandates that the variances of the populations from which the samples are drawn must be approximately equal. Before applying the pooling procedure, statisticians typically perform formal tests, such as Levene's Test or the F-max test, to confirm that this assumption holds true for the data being analyzed.

Ignoring a violation of the homogeneity assumption can lead to serious errors in subsequent hypothesis tests. If the population variances are substantially different (heteroscedasticity), pooling them results in an estimate that is not representative of either population, potentially inflating the Type I error rate (false positives) or reducing the power of the test. Therefore, the preliminary assessment of variance equality is a mandatory step in the statistical workflow before proceeding with the pooled calculation.

When the assumption of equal variances is tenable, the pooling method provides the most efficient and robust estimate of the shared variance. This efficiency arises because we are consolidating information, effectively increasing the sample size used to calculate the measure of dispersion. By ensuring that our foundation--the estimate of variance--is as stable and accurate as possible, we improve the reliability of all subsequent inferential steps, such as calculating confidence intervals or performing a T-test.

The Mathematical Formula for Pooled Variance

For two independent samples, Sample 1 and Sample 2, the formula for the **pooled variance** (s_p^2) is precisely defined to account for the differential influence of sample sizes. Let s_1^2 and s_2^2 be the sample variances, and n_1 and n_2 be the respective sample sizes. The formula mathematically represents the weighted average of the two sample variances, where the weights are $n_1 - 1$ and $n_2 - 1$.

The definitive formula is presented as:

$$s_p^2 = \frac{(n_1 - 1)s_1^2 + (n_2 - 1)s_2^2}{n_1 + n_2 - 2}$$

In this equation, the numerator represents the sum of the squared deviations from the mean for both samples combined, adjusted by their respective degrees of freedom. The term $(n_i - 1)s_i^2$ is algebraically equivalent to the Sum of Squares (SS_i) for that sample. The denominator, $(n_1 + n_2 - 2)$, represents the total degrees of freedom (df) associated with the pooled estimate, which is critical for looking up critical values in statistical tables.

Understanding the components is crucial: s_1^2 and s_2^2 are the standard, unbiased estimates of the population variance derived from each sample. The terms $(n_1 - 1)$ and $(n_2 - 1)$ are the weights based on the sample sizes, ensuring the estimate remains unbiased and gives greater influence to larger samples. The resulting s_p^2 is the most robust estimate of the assumed common population variance ($sigma^2$) under the conditions of homogeneity.

Step-by-Step Calculation Guide

Executing the calculation of **pooled variance** requires careful execution of several sequential steps, whether using raw data or pre-calculated summary statistics. This guide outlines the formal procedure for computing the required inputs and combining them using the established formula.

Calculate Individual Sample Variances: Determine the variance (s_i^2) for each group separately. If starting with raw data, calculate the sample mean ($bar{x}_i$), find the squared deviation of each observation from the mean, sum these squared deviations (Sum of Squares, SS_i), and divide by the degrees of freedom ($n_i - 1$).

Determine Sample Degrees of Freedom: Calculate the degrees of freedom for each sample, $df_1 = n_1 - 1$ and $df_2 = n_2 - 1$. These values will serve as the weights in the numerator.

Calculate Weighted Variances: Multiply each sample variance by its corresponding degrees of freedom: $(n_1 - 1)s_1^2$ and $(n_2 - 1)s_2^2$.

Sum the Weighted Variances: Add the results from the previous step together to find the combined Sum of Squares: $SS_{pooled} = (n_1 - 1)s_1^2 + (n_2 - 1)s_2^2$. This forms the numerator of the pooled variance formula.

Calculate Total Degrees of Freedom: Sum the individual degrees of freedom, which is $df_{total} = (n_1 - 1) + (n_2 - 1)$, or simply $n_1 + n_2 - 2$. This forms the denominator.

Final Calculation: Divide the combined Sum of Squares (Step 4) by the total degrees of freedom (Step 5) to obtain the final pooled variance, s_p^2.

This meticulous process ensures that the resulting pooled estimate is an accurate reflection of the overall dispersion across both datasets, appropriately weighted by the reliability of each sample. It is important to note that the calculation requires the use of sample sizes, not population sizes, reflecting the nature of inferential statistics based on limited observation.

Pooled Variance in the Context of the T-Test

The primary motivation for calculating **pooled variance** is its indispensable role in the **two-sample independent T-test** when the assumption of equal variances is met. When performing this test, we are seeking to determine if the means of two populations are statistically different based on our sample data. The T-statistic formula requires an estimate of the standard error of the difference between the means, and this is where the pooled variance becomes essential.

The standard error of the difference ($mathrm{SE}_{bar{x}_1 - bar{x}_2}$) is calculated using the pooled variance (s_p^2) as follows:

$$\mathrm{SE}_{\bar{x}_1 - \bar{x}_2} = \sqrt{s_p^2 \left(\frac{1}{n_1} + \frac{1}{n_2}\right)}$$

Once the standard error is determined using the robust pooled variance, it is then used in the numerator of the T-statistic formula, which compares the observed difference in sample means to this estimated standard error. Using s_p^2 ensures that the subsequent T-statistic is calculated with the highest possible degrees of freedom, leading to a more powerful test and tighter confidence intervals compared to approaches that do not pool the variance.

Integrating Data Input for Calculation

Calculating the **pooled variance** efficiently often involves specialized statistical software or, as demonstrated below, an interactive tool designed for swift computation. The input required for these tools depends on whether the user possesses the raw data points or has already summarized the data into key statistics.

If you have **raw data**, the tool must internally calculate the means, sum of squares, and individual sample variances (s_1^2 and s_2^2) before proceeding to the pooling formula. If you possess **summary data**--specifically the standard deviation (s_i) or variance (s_i^2) and the sample size (n_i) for each group--the calculation is much faster as the tool bypasses the initial data processing steps. The interactive module below is designed to handle both scenarios seamlessly, providing flexibility in data entry.

To calculate the pooled variance for two samples using this tool, simply select the preferred input method (raw data or summary data) and then accurately fill in the required information. Once the necessary values are entered, clicking the "Calculate" button initiates the script that performs the weighted average calculation based on the provided inputs, resulting in the final robust estimate of the pooled variance (s_p^2).

Interpreting the Pooled Variance Result

The resulting value for the **pooled variance** (s_p^2) provides a measure of the average squared deviation across all data points combined, adjusted for the loss of degrees of freedom due to mean calculation. It is important to remember that variance itself is measured in squared units of the original variable. For practical interpretation, especially when visualizing data dispersion, it is often more intuitive to use the **pooled standard deviation** (s_p), which is simply the square root of s_p^2.

A higher pooled variance indicates greater overall dispersion or variability within the data, while a lower value suggests that the data points tend to cluster more closely around their respective group means. The interpretation is particularly meaningful when compared to the magnitude of the difference between the two sample means. If the mean difference is large relative to the pooled standard deviation, the differences are likely statistically significant.

The calculated pooled variance, such as the example result of 59.905303 provided below, is not merely an abstract number; it is the fundamental scale factor used to standardize the observed difference between means in the T-test. Without this consolidated measure of variability, the test cannot accurately determine the probability that the observed difference occurred merely by chance. Thus, the pooled variance acts as the benchmark against which the hypothesis of equal means is tested.

```
@import url('https://fonts.googleapis.com/css?family=Droid+Serif|Raleway');

.axis--y .domain {
display: none;
}

h1 {
text-align: center;
font-size: 50px;
margin-bottom: 0px;
font-family: 'Raleway', serif;
}

p {
color: black;
margin-bottom: 15px;
margin-top: 15px;
font-family: 'Raleway', sans-serif;
}

#words {
color: black;
font-family: Raleway;
max-width: 550px;
margin: 25px auto;
line-height: 1.75;
padding-left: 100px;
}

#words_calc {
```

```
color: black;
font-family: Raleway;
max-width: 550px;
margin: 25px auto;
line-height: 1.75;
padding-left: 100px;
}

#words_calc input {
display: inline-block;
vertical-align: baseline;
width: 350px;
max-height: 35px;
}

#hr_top {
width: 30%;
margin-bottom: 0px;
border: none;
height: 2px;
color: black;
background-color: black;
}

#hr_bottom {
width: 30%;
margin-top: 15px;
border: none;
height: 2px;
color: black;
background-color: black;
}

#words label, #words input {
display: inline-block;
vertical-align: baseline;
width: 350px;
max-height: 35px;
}

#buttonCalc {
```

```
border: 1px solid;
border-radius: 10px;
margin-top: 20px;
padding: 10px 10px;
cursor: pointer;
outline: none;
background-color: white;
color: black;
font-family: 'Work Sans', sans-serif;
border: 1px solid grey;
/* Green */
}

#buttonCalc:hover {
background-color: #f6f6f6;
border: 1px solid black;
}

#words_output {
text-align: center;
}

#solution_div {
text-align: center;
}

#words_intro {
color: black;
font-family: Raleway;
max-width: 550px;
margin: 25px auto;
line-height: 1.75;
}

#words_table {
color: black;
font-family: Raleway;
max-width: 350px;
margin: 25px auto;
line-height: 1.75;
}
```

```
.text_areas {
color: black;
font-family: Raleway;
max-width: 350px;
margin: 25px auto;
line-height: 1.75;
}

.label_radio {
text-align: center;
}
```

When performing a **two-sample independent t-test**, we typically assume that the variances between the two samples are equal (homogeneity). Under this crucial assumption, we must calculate the **pooled variance** to use as the consolidated estimate of dispersion in the standard error calculation for the T-test.

To calculate the **pooled variance** for two samples, whether utilizing raw observations or pre-summarized data, simply fill in the required information in the sections below and then click the "Calculate" button. This interactive tool provides a quick estimate based on the mathematical principles discussed.

Enter raw data

Enter summary data

Sample 1

301, 298, 295, 297, 304, 305, 309, 298, 291, 299, 293, 304

Sample 2

302, 309, 324, 313, 312, 310, 305, 298, 299, 300, 289, 294

s1 (sample 1 standard deviation)

n1 (sample 1 size)

s2 (sample 2 standard deviation)

n2 (sample 2 size)

Pooled variance = 59.905303

```
//set summary table to hidden to start
var summary_display = document.getElementById("summary_table");
summary_display.style.display = "none";

//find which radio button is checked
function check() {
if (document.getElementById('raw').checked) {
var table_display = document.getElementById("words_table");
table_display.style.display = "block";
var summary_display = document.getElementById("summary_table");
summary_display.style.display = "none";
} else {
var table_display = document.getElementById("words_table");
table_display.style.display = "none";
var summary_display = document.getElementById("summary_table");
summary_display.style.display = "block";
}

} //end check

//perform one-sample t-test
function calc() {
if (document.getElementById('summary').checked) {
var s1 = +document.getElementById('s1').value;
var n1 = +document.getElementById('n1').value;
var s2 = +document.getElementById('s2').value;
var n2 = +document.getElementById('n2').value;

var df = n1-(-1*n2)-2;
var pooled = Math.sqrt(((n1-1)*Math.pow(s1,2) - (-1*((n2-1)*Math.pow(s2,2))))/df);
var pooled2 = pooled*pooled;

document.getElementById('pooled').innerHTML = pooled2.toFixed(6);
} else {
var raw1 = document.getElementById('rawData1').value.split(',').map(Number);
var raw2 = document.getElementById('rawData2').value.split(',').map(Number);
var s1 = math.std(raw1);
var n1 = raw1.length;
var s2 = math.std(raw2);
var n2 = raw2.length;
```

```
var df = n1-(-1*n2)-2;
var pooled = Math.sqrt(((n1-1)*Math.pow(s1,2) - (-1*((n2-1)*Math.pow(s2,2))))/df);
var pooled2 = pooled*pooled;

document.getElementById('pooled').innerHTML = pooled2.toFixed(6);
}

//output results
}
```