

How do you use the maximum likelihood estimation (MLE) method to estimate parameters for a Poisson distribution?

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The maximum likelihood estimation (MLE) method is a statistical technique used to estimate the parameters of a Poisson distribution. This method involves finding the set of parameters that maximizes the likelihood of observing the given data. To use this method, we first need to have a dataset that follows a Poisson distribution. Then, we calculate the likelihood function, which is a measure of how likely the observed data is to occur with different parameter values. Next, using mathematical techniques, we find the values of the parameters that maximize the likelihood function. These values are considered the most likely estimates for the parameters of the Poisson distribution. The MLE method is widely used in various fields, such as mathematics, economics, and biology, to estimate the parameters of a Poisson distribution and make accurate predictions based on the observed data.

MLE for a Poisson Distribution (Step-by-Step)

Maximum likelihood estimation (MLE) is a method that can be used to estimate the parameters of a given distribution.

This tutorial explains how to calculate the MLE for the parameter λ of a Poisson distribution.

Step 1: Write the PDF.

First, write the probability density function of the Poisson distribution:

$$P(X = x) = \frac{\lambda^x e^{-\lambda}}{x!}$$

Step 2: Write the likelihood function.

Next, write the likelihood function. This is simply the product of the PDF for the observed values x_1, \dots, x_n .

$$L(\lambda; x_1, \dots, x_n) = \prod_{j=1}^n \frac{\lambda^{x_j} e^{-\lambda}}{x_j!}$$

Step 3: Write the natural log likelihood function.

To simplify the calculations, we can write the natural log likelihood function:

- $l(\lambda; x_1, \dots, x_n) = \ln \left(\prod_{j=1}^n \frac{\lambda^{x_j} e^{-\lambda}}{x_j!} \right)$
- $l(\lambda; x_1, \dots, x_n) = \sum_{j=1}^n \ln \left(\frac{\lambda^{x_j} e^{-\lambda}}{x_j!} \right)$
- $l(\lambda; x_1, \dots, x_n) = \sum_{j=1}^n [\ln(\lambda^{x_j}) + \ln(e^{-\lambda}) - \ln(x_j!)]$
- $l(\lambda; x_1, \dots, x_n) = \sum_{j=1}^n [x_j \ln(\lambda) - \lambda - \ln(x_j!)]$
- $l(\lambda; x_1, \dots, x_n) = -n\lambda + \ln(\lambda) \sum_{j=1}^n x_j - \sum_{j=1}^n \ln(x_j!)$

Step 4: Calculate the derivative of the natural log likelihood function with respect to λ .

Next, we can calculate the derivative of the natural log likelihood function with respect to the parameter λ :

- $\frac{d}{d\lambda}l(\lambda; x_1, \dots, x_n) = \frac{d}{dy} \left(-n\lambda + \ln(\lambda) \sum_{j=1}^n x_j - \sum_{j=1}^n \ln(x_j!) \right)$
- $\frac{d}{d\lambda}l(\lambda; x_1, \dots, x_n) = -n + \frac{1}{\lambda} \sum_{j=1}^n x_j$

Step 5: Set the derivative equal to zero and solve for λ .

Lastly, we set the derivative in the previous step equal to zero and simply solve for λ :

- $-n + \frac{1}{\lambda} \sum_{j=1}^n x_j = 0$
- $\lambda = \frac{1}{n} \sum_{j=1}^n x_j$

$$\lambda = \frac{1}{n} \sum_{j=1}^n x_j$$

This is equivalent to the sample mean of the n observations in the sample.

An Introduction to the Poisson Distribution

Poisson Distribution Calculator

How to Use the Poisson Distribution in Excel

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