# How do you perform a paired t-test by hand?

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April 29, 2024

A paired t-test is a statistical analysis method used to determine the significance of the difference between two related groups of data. It is typically performed by hand using a series of steps. First, the data for each group is organized and the mean and standard deviation are calculated. Next, the difference between each pair of data values is calculated and a new column is created to represent these differences. The mean, standard deviation, and sample size for the differences are then calculated. Using these values, the t-statistic is computed using the formula (mean of differences / (standard deviation of differences / square root of sample size)). This t-statistic is then compared to the critical t-value for the chosen level of significance, typically 0.05. If the t-statistic is greater than the critical t-value, the difference between the two groups is considered to be significant. Finally, the results are interpreted and conclusions are drawn regarding the significance of the difference between the two groups.

## Perform a Paired t-Test by Hand

## A is used to compare the means of two samples when each in one sample can be paired with an observation in the other sample.

## The following step-by-step example shows how to perform a paired samples t-test to determine if the population means are equal between the following two groups:

| Group 1 | Group 2 |
|---|---|
| 13 | 9 |
| 14 | 11 |
| 14 | 12 |
| 15 | 12 |
| 16 | 14 |
| 17 | 16 |
| 17 | 18 |
| 18 | 18 |
| 19 | 18 |
| 20 | 19 |
| 22 | 20 |
| 23 | 20 |

### Step 1: Calculate the Test Statistic

**The test statistic of a paired t-test is calculated as:**

**t = xdiff / (sdiff/√n)**

**where:**

**xdiff: sample mean of the differencess: sample standard deviation of the differencesn: sample size (i.e. number of pairs)**

**We will calculate the mean of the differences between the two groups and the standard deviation of the differences between the two groups:**

| Group 1 | Group 2 | Difference |
|---|---|---|
| 13 | 9 | 4 |
| 14 | 11 | 3 |
| 14 | 12 | 2 |
| 15 | 12 | 3 |
| 16 | 14 | 2 |
| 17 | 16 | 1 |
| 17 | 18 | -1 |
| 18 | 18 | 0 |
| 19 | 18 | 1 |
| 20 | 19 | 1 |
| 22 | 20 | 2 |
| 23 | 20 | 3 |
| Mean of differences | | **1.75** |
| Std. Dev. of differences | | **1.422** |



**Thus, our test statistic can be calculated as:**

**t = xdiff / (sdiff/√n)t = 1.75 / (1.422/√12)t = 4.26**

### Step 2: Calculate the Critical Value

**Next, we need to find the critical value to compare our test statistic to.**

**For this example, we'll use a two-tailed test with $\alpha$ = .05 and df = n-1 degrees of freedom.**

**According to the , the critical value that corresponds to these values is 2.201:**

| | P | | | | | | |
|---|---|---|---|---|---|---|---|
| one-tail | 0.1 | 0.05 | 0.025 | 0.01 | 0.005 | 0.001 | 0.0005 |
| two-tails | 0.2 | 0.1 | 0.05 | 0.02 | 0.01 | 0.002 | 0.001 |
| DF | | | | | | | |
| 1 | 3.078 | 6.314 | 12.706 | 31.821 | 63.656 | 318.289 | 636.578 |
| 2 | 1.886 | 2.92 | 4.303 | 6.965 | 9.925 | 22.328 | 31.6 |
| 3 | 1.638 | 2.353 | 3.182 | 4.541 | 5.841 | 10.214 | 12.924 |
| 4 | 1.533 | 2.132 | 2.776 | 3.747 | 4.604 | 7.173 | 8.61 |
| 5 | 1.476 | 2.015 | 2.571 | 3.365 | 4.032 | 5.894 | 6.869 |
| 6 | 1.44 | 1.943 | 2.447 | 3.143 | 3.707 | 5.208 | 5.959 |
| 7 | 1.415 | 1.895 | 2.365 | 2.998 | 3.499 | 4.785 | 5.408 |
| 8 | 1.397 | 1.86 | 2.306 | 2.896 | 3.355 | 4.501 | 5.041 |
| 9 | 1.383 | 1.833 | 2.262 | 2.821 | 3.25 | 4.297 | 4.781 |
| 10 | 1.372 | 1.812 | 2.228 | 2.764 | 3.169 | 4.144 | 4.587 |
| 11 | 1.363 | 1.796 | 2.201 | 2.718 | 3.106 | 4.025 | 4.437 |
| 12 | 1.356 | 1.782 | 2.179 | 2.681 | 3.055 | 3.93 | 4.318 |
| 13 | 1.35 | 1.771 | 2.16 | 2.65 | 3.012 | 3.852 | 4.221 |
| 14 | 1.345 | 1.761 | 2.145 | 2.624 | 2.977 | 3.787 | 4.14 |
| 15 | 1.341 | 1.753 | 2.131 | 2.602 | 2.947 | 3.733 | 4.073 |
| 16 | 1.337 | 1.746 | 2.12 | 2.583 | 2.921 | 3.686 | 4.015 |
| 17 | 1.333 | 1.74 | 2.11 | 2.567 | 2.898 | 3.646 | 3.965 |
| 18 | 1.33 | 1.734 | 2.101 | 2.552 | 2.878 | 3.61 | 3.922 |
| 19 | 1.328 | 1.729 | 2.093 | 2.539 | 2.861 | 3.579 | 3.883 |
| 20 | 1.325 | 1.725 | 2.086 | 2.528 | 2.845 | 3.552 | 3.85 |



### Step 3: Reject or Fail to Reject the Null Hypothesis

**Our paired samples t-test uses the following null and alternative hypothesis:**

**H0: μ1 = μ2 (the two population means are equal)HA: μ1 ? μ2 (the two population means are not equal)**

**Since the absolute value of our test statistic (4.26) is greater than the critical value found in the t-table (2.201), we reject the null hypothesis.**

**This means we have sufficient evidence to say that the mean between the two groups is not equal.**

**Bonus: Feel free to use the to confirm your results.**