

# How do you calculate upper and lower fence?

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## RECOMMENDED CITATION

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```
@import url('https://fonts.googleapis.com/css?family=Droid+Serif|Raleway');
```

```
h1 {  
text-align: center;  
font-size: 50px;  
margin-bottom: 0px;  
font-family: 'Raleway', serif;  
}
```

```
p {  
color: black;  
margin-bottom: 15px;  
margin-top: 15px;  
font-family: 'Raleway', sans-serif;  
}
```

```
#words {  
padding-left: 30px;  
color: black;  
font-family: Raleway;  
max-width: 550px;  
margin: 25px auto;  
line-height: 1.75;  
}
```

```
#words_summary {  
padding-left: 70px;  
color: black;  
font-family: Raleway;  
max-width: 550px;  
margin: 25px auto;  
line-height: 1.75;  
}
```

```
#words_text {  
color: black;  
font-family: Raleway;  
max-width: 550px;  
margin: 25px auto;  
line-height: 1.75;  
}
```

```
#words_text_area {  
display:inline-block;  
color: black;  
font-family: Raleway;  
max-width: 550px;  
margin: 25px auto;  
line-height: 1.75;  
padding-left: 100px;  
}
```

```
#calcTitle {  
text-align: center;  
font-size: 20px;  
margin-bottom: 0px;  
font-family: 'Raleway', serif;  
}
```

```
#hr_top {  
width: 30%;  
margin-bottom: 0px;  
border: none;  
height: 2px;  
color: black;  
background-color: black;  
}
```

```
#hr_bottom {  
width: 30%;  
margin-top: 15px;  
border: none;  
height: 2px;  
color: black;  
background-color: black;  
}
```

```
#words_table label, #words_table input {  
display: inline-block;  
vertical-align: baseline;  
width: 350px;  
}
```

```
#buttonCalc {  
border: 1px solid;  
border-radius: 10px;  
margin-top: 20px;  
  
cursor: pointer;  
outline: none;  
background-color: white;  
color: black;  
font-family: 'Work Sans', sans-serif;  
border: 1px solid grey;  
/* Green */  
}
```

```
#buttonCalc:hover {  
background-color: #f6f6f6;  
border: 1px solid black;  
}
```

```
#words_table {  
color: black;  
font-family: Raleway;  
max-width: 350px;  
margin: 25px auto;  
line-height: 1.75;  
}
```

```
#summary_table {  
color: black;  
font-family: Raleway;  
max-width: 550px;  
margin: 25px auto;  
line-height: 1.75;  
padding-left: 20px;  
}
```

```
.label_radio {  
text-align: center;  
}
```

```
td, tr, th {
```

```
border: 1px solid black;
}
table {
border-collapse: collapse;
}
td, th {
min-width: 50px;
height: 21px;
}
.label_radio {
text-align: center;
}

#text_area_input {
padding-left: 35%;
float: left;
}

svg:not(:root) {
overflow: visible;
}
```

## 1. Introduction: Understanding Statistical Fences

In the field of statistics, establishing clear boundaries is essential for accurate data analysis and interpretation. The concept of the **upper and lower fences** provides analysts with a standardized, robust method for identifying potential outliers within a given dataset. These fences act as cut-off values, delineating the range where typical data points are expected to reside. Data points that fall outside these limits are statistically considered unusual, warranting closer scrutiny to determine if they represent legitimate extreme values or errors in measurement or recording. Understanding how to calculate and apply these fences is fundamental for data cleaning and preparing data for more advanced modeling.

The method used to calculate these fences is derived from the five-number summary and relies heavily on the distribution of the central 50% of the data. This technique offers a less sensitive measure compared to methods based on the mean and standard deviation, which can be heavily influenced by the presence of extreme values themselves. By using percentiles, specifically the quartiles, the fence calculation maintains its resilience against distributional anomalies. The consistent application of the fence method ensures that data quality is maintained across various analyses, leading to more reliable scientific conclusions and business insights. This article will

provide an in-depth guide on the methodology behind calculating these critical statistical boundaries.

## 2. The Foundation: Quartiles (Q1 and Q3)

Before calculating the fences, we must first establish the positions of the key internal reference points: the quartiles. Quartiles divide an ordered dataset into four equal sections. The second quartile (Q2) is simply the median, which splits the data into two halves. However, for fence calculation, we are primarily concerned with the first quartile (**Q1**) and the third quartile (**Q3**).

The first quartile, or **Q1**, represents the 25th percentile. When a dataset is sorted from smallest to largest, Q1 is the value below which 25% of the data points lie. Conversely, the third quartile, or **Q3**, represents the 75th percentile. This is the value below which 75% of the observations fall, meaning 25% of the data points are greater than Q3. Accurate calculation of these quartiles is the first and most critical step, as any error here will propagate throughout the subsequent steps of determining the upper and lower fences. Various methods exist for calculating quartiles, especially when the number of data points is not perfectly divisible by four, but all aim to find the points that truly divide the data distribution.

In practical terms, the quartiles help us understand the spread and skewness of the central portion of the data. If the distance between Q1 and the median (Q2) is significantly different from the distance between Q3 and the median, it suggests that the data distribution is skewed. This foundational understanding allows the calculation of the **Interquartile Range (IQR)**, which measures the span of the middle half of the observations, forming the essential building block for defining the fence limits.

## 3. Defining the Interquartile Range (IQR)

The **Interquartile Range (IQR)** is a measure of statistical dispersion, representing the difference between the third quartile (**Q3**) and the first quartile (**Q1**). It is fundamentally defined by the range of the middle 50% of the observations in the dataset. Unlike the total range (maximum value minus minimum value), the **IQR** is highly resistant to the influence of extreme values because it excludes the lowest 25% and the highest 25% of the data points. This resistance is precisely why it is employed in the calculation of statistical fences.

The formula for the **IQR** is straightforward:

$$\text{IQR} = \text{Q3} - \text{Q1}$$

A small **IQR** indicates that the central 50% of the data points are clustered closely together, suggesting low variability in the typical values. Conversely, a large **IQR** implies greater spread

among the core observations. This value of spread, scaled by a fixed multiplier, determines the distance the fences are set away from the central quartiles. The IQR, therefore, acts as a dynamic unit of measurement based on the intrinsic variability of the data itself, ensuring that the boundaries for identifying outliers are appropriate for that specific distribution.

It is important to recognize that the IQR is the cornerstone of the box plot (or box-and-whisker plot) visualization, where the box itself spans from Q1 to Q3. By using the IQR as the basis for the fence calculation, we ensure that the definition of an outlier is contextually relevant to the concentration of the bulk of the data, thereby providing a reliable method for identifying values that truly deviate significantly from the norm.

#### 4. The Role of the 1.5 Multiplier Rule

The definition of an outlier requires a universally accepted criterion that balances sensitivity (catching true anomalies) and specificity (avoiding false positives). This balance is achieved through the use of the **1.5 multiplier rule**, a standard convention established in robust statistics. The factor of 1.5 is multiplied by the Interquartile Range (IQR) to determine the distance that the fences should extend beyond the quartiles (Q1 and Q3).

The choice of 1.5 is conventional and historical, originating largely from John Tukey's work on exploratory data analysis. While not derived from a specific probability distribution like the standard deviation rules (e.g., the 3-sigma rule), the 1.5 factor has proven empirically effective across a wide variety of data distributions. When applied to a perfectly normal distribution, this rule typically identifies observations that fall outside approximately 2.7 standard deviations from the mean, covering about 99.3% of the data points. Therefore, points outside the fences—the potential outliers—are those that occur in the extreme tails of the distribution.

The calculated value, 1.5 multiplied by the IQR, is often referred to as the outlier threshold. This threshold is then subtracted from Q1 to determine the **Lower Fence** and added to Q3 to determine the **Upper Fence**. This scalable approach ensures that whether the data is highly dispersed or tightly packed, the definition of an extreme value remains proportional to the data's central variation. This methodology effectively safeguards against the influence of the outliers themselves skewing the very measure used to identify them, making the fence calculation a robust statistical measure.

#### 5. Formulas for the Upper and Lower Fences

The complete calculation process combines the foundational quartile values (Q1 and Q3) with the variability metric (IQR) scaled by the conventional 1.5 multiplier. These two resulting boundary values—the **Upper Fence** and the **Lower Fence**—establish the official thresholds for outlier detection within the dataset.

The formulas are precisely defined as follows:

**Upper Fence (UF):** This boundary is calculated by adding the scaled IQR to the third quartile (Q3). Any data point greater than this value is considered a potential upper outlier.

$$\text{Upper fence} = Q3 + (1.5 \times \text{IQR})$$

**Lower Fence (LF):** This boundary is calculated by subtracting the scaled IQR from the first quartile (Q1). Any data point less than this value is considered a potential lower outlier.

$$\text{Lower fence} = Q1 - (1.5 \times \text{IQR})$$

It is important to note that the fences themselves are not necessarily data points within the dataset; they are calculated statistical thresholds. Once these thresholds are computed, every data point must be compared against them. If a value 'x' satisfies the condition  $x > \text{UF}$  or  $x < \text{LF}$ , that value is flagged for review as an extreme observation. These straightforward formulas provide a powerful, non-parametric method for data quality control.

## 6. Practical Application: Step-by-Step Calculation Example

To solidify the understanding of fence calculation, let us walk through a detailed example using the sample dataset provided in the interactive calculator below. Assume our ordered dataset is: 11, 13, 14, 14, 15, 16, 18, 22, 24, 27, 34, 36, 38, 41, 45. There are  $N=15$  observations.

**Step 1: Calculate Q1 and Q3.** Since there are 15 observations, the median (Q2) is the 8th observation, which is 22. The lower half of the data is 11, 13, 14, 14, 15, 16, 18. Q1 is the median of this lower half, which is the 4th observation, 14. The upper half of the data is 24, 27, 34, 36, 38, 41, 45. Q3 is the median of this upper half, which is the 4th observation in that set, 36. Thus, **Q1 = 14.00** and **Q3 = 36.00**.

**Step 2: Calculate the Interquartile Range (IQR).** The IQR is the difference between Q3 and Q1.  $\text{IQR} = 36.00 - 14.00 = 22.00$ . This value of 22 represents the spread of the central 50% of the data points in this specific distribution. This IQR will now be used to determine the necessary buffer distance for the fences.

**Step 3: Calculate the Lower Fence (LF).** We apply the formula  $\text{LF} = Q1 - (1.5 \times \text{IQR})$ .  $\text{LF} = 14.00 - (1.5 \times 22.00)$ . Since  $1.5 \times 22.00$  equals 33.00, we subtract this from Q1:  $14.00 - 33.00 = -19.00$ . Any value below -19.00 would be flagged as a lower outlier. Since the smallest value in our dataset is 11, there are no lower outliers in this specific example.

**Step 4: Calculate the Upper Fence (UF).** We apply the formula  $\text{UF} = Q3 + (1.5 \times \text{IQR})$ .  $\text{UF} = 36.00 + (1.5 \times 22.00)$ . Adding 33.00 to Q3 yields:  $36.00 + 33.00 = 69.00$ . Any value above 69.00

would be flagged as an upper outlier. Since the largest value in our dataset is 45, there are no upper outliers in this specific example either. The calculator below demonstrates these exact calculations interactively.

## 7. Visualizing Fences: The Box Plot Connection

The calculation of the fences is intrinsically linked to the construction of a **Box-and-Whisker Plot**, often simply called a box plot. The box plot is one of the most effective tools in exploratory data analysis (EDA) for summarizing the distribution of a dataset, showing the median, quartiles, and most importantly, the outliers identified by the fences. The central box spans from Q1 to Q3 (the IQR), and the median (Q2) is marked inside the box.

The whiskers of the box plot extend outwards from the box to the most extreme data points that are still within the calculated fence limits. That is, the upper whisker extends to the maximum value that is less than or equal to the **Upper Fence**, and the lower whisker extends to the minimum value that is greater than or equal to the **Lower Fence**. If the maximum or minimum data points fall within the fences, the whiskers simply end there. If the maximum value is greater than the Upper Fence, that value is plotted separately as an outlier, and the whisker stops at the next largest observation that is still inside the fence.

Any data points lying outside the range defined by the whiskers—i.e., beyond the upper or lower fences—are traditionally marked by distinct symbols, such as asterisks or circles, highlighting their status as potential outliers. This visualization makes the impact of the fence calculation immediately apparent, allowing analysts to quickly assess the spread, symmetry, and presence of extreme values in the data distribution. The box plot provides a compelling graphical representation of the quantitative results derived from the fence calculation method.

## 8. Implications of Outliers and Data Cleaning

The primary purpose of calculating the **upper and lower fences** is to systematically identify potential outliers. Once a data point is identified as falling outside these established boundaries, it triggers a crucial phase of data analysis known as scrutiny or data cleaning. An outlier is not necessarily an error, but its existence requires investigation. Outliers can arise from genuine, albeit rare, events in the phenomena being measured, or they can be the result of recording errors, measurement failures, or equipment malfunctions.

If an outlier is determined to be the result of an error (e.g., a typo where 100 was entered instead of 10), it should be corrected or removed from the analysis, as its inclusion could drastically distort statistical measures such as the mean and standard deviation, leading to incorrect conclusions. However, if the outlier represents a genuine extreme observation (e.g., a record-breaking stock price or an unusually high customer purchase), its removal must be justified cautiously.

Sometimes, these legitimate outliers hold the most valuable information, revealing critical insights or new patterns. Decisions regarding the handling of outliers should be documented transparently as part of the statistical methodology.

In summary, the fence method provides a standardized, quantifiable metric—the 1.5 IQR rule—that moves the identification of extreme values beyond mere visual inspection or subjective judgment. By establishing these objective thresholds, analysts ensure consistency and rigor in their data preprocessing pipeline, improving the robustness and reliability of all subsequent statistical modeling and inferences drawn from the dataset.

The following interactive tool allows you to input your own comma-separated dataset and immediately calculate the critical statistical values required to determine the upper and lower fences:

**Values:**

11, 13, 14, 14, 15, 16, 18, 22, 24, 27, 34, 36, 38, 41, 45

Calculate Fences

Q1 (First Quartile) = 14.00

Q3 (Third Quartile) = 36.00

IQR (Interquartile Range) = 22.00

Lower Fence Calculation:  $14.00 - (1.5 \times 22.00) = -19.00$

Upper Fence Calculation:  $36.00 + (1.5 \times 22.00) = 69.00$

```
function calc() {  
  
  //get input data  
  var x= document.getElementById('x').value.split(',').map(Number);  
  
  //calculate stuff  
  var _median = math.median(x)  
  var _firstHalf = x.filter(function(f){ return f < _median })  
  
  var Q1 = math.median(_firstHalf);  
  var Q3 = math.median(_secondHalf);  
  var IQR = Q3 - Q1;  
  
  var lower = Q1 - (1.5*IQR);
```

```
var upper = Q3 -(-1*1.5*IQR);

//output stuff
document.getElementById('Q1').innerHTML = Q1.toFixed(2);
document.getElementById('Q3').innerHTML = Q3.toFixed(2);
document.getElementById('IQR').innerHTML = IQR.toFixed(2);
document.getElementById('lower').innerHTML = lower.toFixed(2);
document.getElementById('upper').innerHTML = upper.toFixed(2);

document.getElementById('Q1_out').innerHTML = Q1.toFixed(2);
document.getElementById('IQR_out1').innerHTML = IQR.toFixed(2);
document.getElementById('Q3_out').innerHTML = Q3.toFixed(2);
document.getElementById('IQR_out2').innerHTML = IQR.toFixed(2);

} //end calc function
```