

How do you calculate a Poisson confidence interval step-by-step?

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A Poisson confidence interval is a statistical method used to estimate the true mean or average of a Poisson-distributed variable with a specified level of confidence. This interval can be calculated using the following steps:

1. Determine the sample size (n) and the observed number of events (x).
2. Calculate the sample mean (\bar{x}) by dividing the observed number of events by the sample size ($\bar{x} = x / n$).
3. Calculate the standard error (SE) by taking the square root of the sample mean ($SE = \sqrt{\bar{x}}$).
4. Determine the desired confidence level (c) in decimal form, typically 95% or 99%.
5. Look up the corresponding critical value (z^*) for the chosen confidence level in a standard normal distribution table.
6. Calculate the lower and upper bounds of the confidence interval using the following formulas:

$$\text{Lower bound} = \bar{x} - (z^* * SE)$$

$$\text{Upper bound} = \bar{x} + (z^* * SE)$$

7. The resulting interval represents the estimated range in which the true mean lies with the chosen confidence level.

Calculate a Poisson Confidence Interval (Step-by-Step)

The is a probability distribution that is used to model the probability that a certain number of events occur during a fixed time interval when the events are known to occur independently and with a constant mean rate.

While it's helpful to know the mean number of occurrences of some Poisson process, it can be even more helpful to have a around the mean number of occurrences.

For example, suppose we collect data at a call center on

a random day and find that the mean number of calls per hour is 15.

Since we only collected data on one single day, we can't be certain that the call center receives 15 calls per hour, on average, throughout the entire year.

However, we can use the following formula to calculate a confidence interval for the mean number of calls per hour:

Poisson Confidence Interval Formula

Confidence Interval =

where:

X²: Chi-Square Critical Value
N: The number of observed events
α: The significance level

The following step-by-step example illustrates how to calculate a 95% Poisson confidence interval in practice.

Step 1: Count the Observed Events

Suppose we calculate the mean number of calls per

hour at a call center to be 15. Thus, $N = 15$.

And since we're calculating a 95% confidence interval, we'll use $\alpha = .05$ in the following calculations.

Step 2: Find the Lower Confidence Interval Bound

The lower confidence interval bound is calculated as:

Lower bound = $0.5 * X_{22N, \alpha/2}$
 Lower bound = $0.5 * X_{22(15), .975}$
 Lower bound = $0.5 * X_{230, .975}$
 Lower bound = $0.5 * 16.791$
 Lower bound = 8.40

Note: We used the to compute $X_{230, .975}$.

Step 3: Find the Upper Confidence Interval Bound

The upper confidence interval bound is calculated as:

Upper bound = $0.5 * X_{22(N+1), 1-\alpha/2}$
 Upper bound = $0.5 * X_{22(15+1), .025}$
 Upper bound = $0.5 * X_{232, .025}$
 Upper bound = $0.5 * 49.48$
 Upper bound = 24.74

Step 4: Find the Confidence Interval

Using the lower and upper bounds previously computed, our 95% Poisson confidence interval turns out to be:

95% C.I. =

This means we are 95% confident that the true mean number of calls per hour that the call center receives is between 8.40 calls and 24.74 calls.

Bonus: Poisson Confidence Interval Calculator

Feel free to use this to automatically compute a Poisson confidence interval.

For example, here's how to use this calculator to find the Poisson confidence interval we just computed manually:

Observed Events

Confidence Level

CALCULATE

95% Confidence Interval = [8.39539, 24.74022]

Notice that the results match the confidence interval that we computed manually.