# How can a Two-Way ANOVA be performed by hand?

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A Two-Way ANOVA, also known as a two-factor ANOVA, is a statistical method used to analyze the effects of two independent variables on a dependent variable. This analysis can be performed by hand using a series of calculations and formulas. First, the data must be organized into a table, with the dependent variable in one column and the two independent variables in separate columns. Next, the means for each variable must be calculated. Then, the total sum of squares, treatment sum of squares, and error sum of squares must be calculated using specific formulas. These values are then used to calculate the F-ratio, which is compared to a critical value to determine if there is a significant difference between the groups. The process is repeated for each variable, and the results are interpreted to determine the effects of each independent variable on the dependent variable. While this method can be time-consuming, it allows for a detailed and thorough analysis of the data.

## Perform a Two-Way ANOVA by Hand

## A is used to determine whether or not there is a statistically significant difference between the means of three or more independent groups that have been split on two factors.

## This tutorial explains how to perform a two-way ANOVA by hand.

### Example: Two-Way ANOVA by Hand

## Suppose a botanist wants to know if plant growth is influenced by sunlight exposure and watering frequency. She plants 40 seeds and lets them grow for one month under different conditions for sunlight exposure and watering frequency.

**After one month, she records the height of each plant. The results are shown below:**

| | Sunlight Exposure | | | |
|---|---|---|---|---|
| **Watering Frequency** | None | Low | Medium | High |
| Daily | 4.8 | 5 | 6.4 | 6.3 |
| | 4.4 | 5.2 | 6.2 | 6.4 |
| | 3.2 | 5.6 | 4.7 | 5.6 |
| | 3.9 | 4.3 | 5.5 | 4.8 |
| | 4.4 | 4.8 | 5.8 | 5.8 |
| Weekly | 4.4 | 4.9 | 5.8 | 6 |
| | 4.2 | 5.3 | 6.2 | 4.9 |
| | 3.8 | 5.7 | 6.3 | 4.6 |
| | 3.7 | 5.4 | 6.5 | 5.6 |
| | 3.9 | 4.8 | 5.5 | 5.5 |



**In the table above, we see that there were five plants grown under each combination of conditions.**

**For example, there were five plants grown with daily watering and no sunlight and their heights after two months were 4.8 inches, 4.4 inches, 3.2 inches, 3.9 inches, and 4.4 inches:**

| | Sunlight Exposure | | | |
|---|---|---|---|---|
| Watering Frequency | None | Low | Medium | High |
| Daily | 4.8 | 5 | 6.4 | 6.3 |
| | 4.4 | 5.2 | 6.2 | 6.4 |
| | 3.2 | 5.6 | 4.7 | 5.6 |
| | 3.9 | 4.3 | 5.5 | 4.8 |
| | 4.4 | 4.8 | 5.8 | 5.8 |
| Weekly | 4.4 | 4.9 | 5.8 | 6 |
| | 4.2 | 5.3 | 6.2 | 4.9 |
| | 3.8 | 5.7 | 6.3 | 4.6 |
| | 3.7 | 5.4 | 6.5 | 5.6 |
| | 3.9 | 4.8 | 5.5 | 5.5 |

**We can use the following steps to perform a two-way ANOVA:**



## Step 1: Calculate Sum of Squares for First Factor (Watering Frequency)

**First, we will calculate the grand mean height of all 40 plants:**

**Grand mean = (4.8 + 5 + 6.4 + 6.3 + ... + 3.9 + 4.8 + 5.5 + 5.5) / 40 = 5.1525**

**Next, we will calculate the mean height of all plants watered daily:**

**Mean of Daily = (4.8 + 5 + 6.4 + 6.3 + ... + 4.4 + 4.8 + 5.8 + 5.8) /20 = 5.155**

**Next, we will calculate the mean height of all plants watered weekly:**

**Mean of Weekly = (4.4 + 4.9 + 5.8 + 6 + ... + 3.9 + 4.8 + 5.5 + 5.5) /20 = 5.15**

**Σn(Xj - X..)2**

**where:**

**n: the sample size of group jΣ: a greek symbol that means "sum"Xj: the mean of group jX..: the grand mean**

**In our example, we calculate the sum of squares for the factor "watering frequency" to be: 20(5.155-5.1525)2 + 20(5.15-5.1525)2 = .00025**

**Step 2: Calculate Sum of Squares for Second Factor (Sunlight Exposure)**

**First, we will calculate the grand mean height of all 40 plants:**

**Grand mean = (4.8 + 5 + 6.4 + 6.3 + ... + 3.9 + 4.8 + 5.5 + 5.5) / 40 = 5.1525**

**Next, we will calculate the mean height of all plants with**

**no sunlight exposure:**

**Mean of No Sunlight = (4.8 + 4.4 + 3.2 + 3.9 + 4.4 + 4.4 + 4.2 + 3.8 + 3.7 + 3.9) / 10 = 4.07**

**We will repeat this calculation to find the mean height of plants with various sunlight exposures:**

**Mean of Low Sunlight = 5.1Mean of Medium Sunlight = 5.89Mean of High Sunlight = 5.55**

**Next, we will calculate the sum of squares for the factor "sunlight exposure" by using the following formula:**

**Σn(Xj - X..)2**

**where:**

**n: the sample size of group jΣ: a greek symbol that means "sum"Xj: the mean of group jX..: the grand mean**

**In our example, we calculate the sum of squares for the factor "sunlight exposure" to be: 10(4.07-5.1525)2 + 10(5.1-5.1525)2 + 10(5.89-5.1525)2 + 10(5.55-5.1525)2 = 18.76475**

**Step 3: Calculate Sum of Squares Within (Error)**

**Next, we will calculate the sum of squares within by taking the sum of squared differences between each combination of factors and the individual plant heights.**

**For example, the mean height of all plants watered daily with no sunlight exposure is 4.14. We can then calculate the sum of squared differences for each of these individual plants as:**

**SS for daily watering and no sunlight: (4.8-4.14)2 + (4.4-4.14)2 + (3.2-4.14)2 + (3.9-4.14)2 + (4.4-4.14)2 = 1.512**

**We can repeat this process for each combination of factors:**

**SS for daily watering and low sunlight: 0.928SS for daily watering and medium sunlight: 1.788SS for daily watering and high sunlight: 1.648SS for weekly watering and no sunlight: 0.34SS for weekly watering and low sunlight: 0.548SS for weekly watering and medium sunlight: 0.652SS for weekly watering and high sunlight: 1.268**

**We can then take the sum of all of these values to find the sum of squares within (error):**

**Sums of squares within = 1.512 + .928 + 1.788 + 1.648 + .34 + .548 + .652 + 1.268 = 8.684**

**Step 4: Calculate Total Sum of Squares**

**Next, we can calculate the total sum of squares by taking the sum of the differences between each individual plant height and the grand mean:**

**Total Sum of Squares = (4.8 - 5.1525)2 + (5 - 5.1525)2 + ... + (5.5 - 5.1525)2 = 28.45975**

**Step 5: Calculate Sum of Squares Interaction**

**Next, we will calculate the sum of squares interaction by using the following formula:**

**SS Interaction = SS Total - SS Factor 1 - SS Factor 2 - SS WithinSS Interaction = 28.45975 - .00025 - 18.76475 - 8.684SS Interaction = 1.01075**

**Step 6: Fill in ANOVA Table**

**Lastly, we'll fill in the values for the two-way ANOVA table:**

| Source of Variation | SS | df | MS | F | p-value |
|---|---|---|---|---|---|
| Watering Frequency | 0.00025 | 1 | 0.00025 | 0.000921 | 0.975 |
| Sunlight Exposure | 18.76475 | 3 | 6.254917 | 23.04898 | <.000 |
| Interaction | 1.01075 | 3 | 0.336917 | 1.241517 | 0.311 |
| Within | 8.684 | 32 | 0.271375 | | |
| | | | | | |
| Total | 28.45975 | 39 | | | |

## Here is how we calculated the various numbers in the table:

## df Watering Frequency: j-1 = 2-1 = 1df Sunlight Exposure: k-1 = 4-1 = 3df Interaction: (j-1)*(k-1) = 1*3 = 3df Within: n - (j*k) = 40 - (2*4) = 32df total: n-1 = 40-1 = 39MS: SS/ dfF Watering Frequency: MS Watering Frequency / MS WithinF Sunlight Exposure: MS Sunlight Exposure / MS WithinF Interaction: MS Interaction / MS Withinp-value Watering Frequency: The p-value that corresponds to F value of .000921 with df numerator = 1 and df denominator = 32p-value Sunlight Exposure: The p-value that corresponds to F value of 23.04898 with df numerator = 3 and df denominator = 32p-value Interaction: The p-value that corresponds to F value of 1.241517 with df numerator = 3 and df denominator = 32

**Note #1: n = total observations, j = number of levels for watering frequency, k = number of levels for sunlight exposure.**

**Note #2: The p-values that correspond to the F-value were calculated using the .**

**Step 7: Interpret the results**

**We can observe the following from the ANOVA table:**

**The p-value for the interaction between watering frequency and sunlight exposure was 0.311. This is not statistically significant at α = 0.05.The p-value for watering frequency was 0.975. This is not statistically significant at α = 0.05.The p-value for sunlight exposure was < 0.000. This is statistically significant at α = 0.05.**

**These results indicate that sunlight exposure is the only factor that has a statistically significant effect on plant height.**

**And because there is no interaction effect, the effect of sunlight exposure is consistent across each level of watering frequency.**

## That is, whether a plant is watered daily or weekly has no impact on how sunlight exposure affects a plant.

### Additional Resources

## The following tutorials provide additional information about ANOVA's: