

How to Easily Understand the Difference Between Normal and Standard Normal Distributions

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The field of statistics relies heavily on understanding how data is distributed, and few concepts are as fundamental as the Normal Distribution. This foundational statistical model, often colloquially referred to as the bell curve, describes a probability distribution applicable to a vast range of continuous random variables observed in nature and research. A key identifying characteristic of this distribution is its perfect symmetry, meaning the **mean**, **median**, and **mode** all coincide at the exact center of the curve. This symmetry allows for powerful statistical inferences and modeling capabilities.

While the Normal Distribution describes a family of distributions defined by their unique mean (location) and standard deviation (spread), the **Standard Normal Distribution** represents a highly specific and critical instance within this family. It is precisely defined by having a mean (μ) equal to 0 and a standard deviation (σ) equal to 1. This standardized structure makes it an invaluable tool for comparison and calculation across diverse datasets. Because this specific distribution is derived through the calculation of Z-scores, it is frequently referred to as the **Z-distribution**, serving as the benchmark for standardized statistical analysis.

The standardization achieved by the Standard Normal Distribution allows statisticians to universally compare observations from vastly different datasets. The underlying mechanism for this comparison is the Z-score, which quantifies the distance, measured in units of standard deviations, that a particular data point lies away from its distribution's mean. By translating any normal data value into a Z-score, we effectively place it onto the common scale of the Standard Normal Distribution, enabling direct comparison regardless of the original units or scales of measurement.

Understanding the Universal Nature of the Normal Distribution

The Normal Distribution is undeniably the most frequently utilized and analyzed probability distribution across all branches of quantitative analysis, from finance and engineering to biology and social sciences. Its prominence stems largely from the **Central Limit Theorem**, which asserts that the distribution of sample means approximates a normal distribution as the sample size increases, regardless of the population's original distribution. This fundamental principle makes the Normal Distribution essential for hypothesis testing and confidence interval construction, justifying its common use in nearly all statistical methodologies.

The structural characteristics of the bell curve are mathematically consistent and statistically significant. The key features that define a Normal Distribution are straightforward yet powerful. Firstly, it must be perfectly **symmetrical** around its central point. Secondly, the distribution is uniquely defined by its two parameters: the **mean** (μ) and the **standard deviation** (σ). The mean dictates the central location of the peak of the curve, while the standard deviation governs the curve's overall spread or dispersion.

We can summarize the intrinsic properties of any Normal Distribution through the following points,

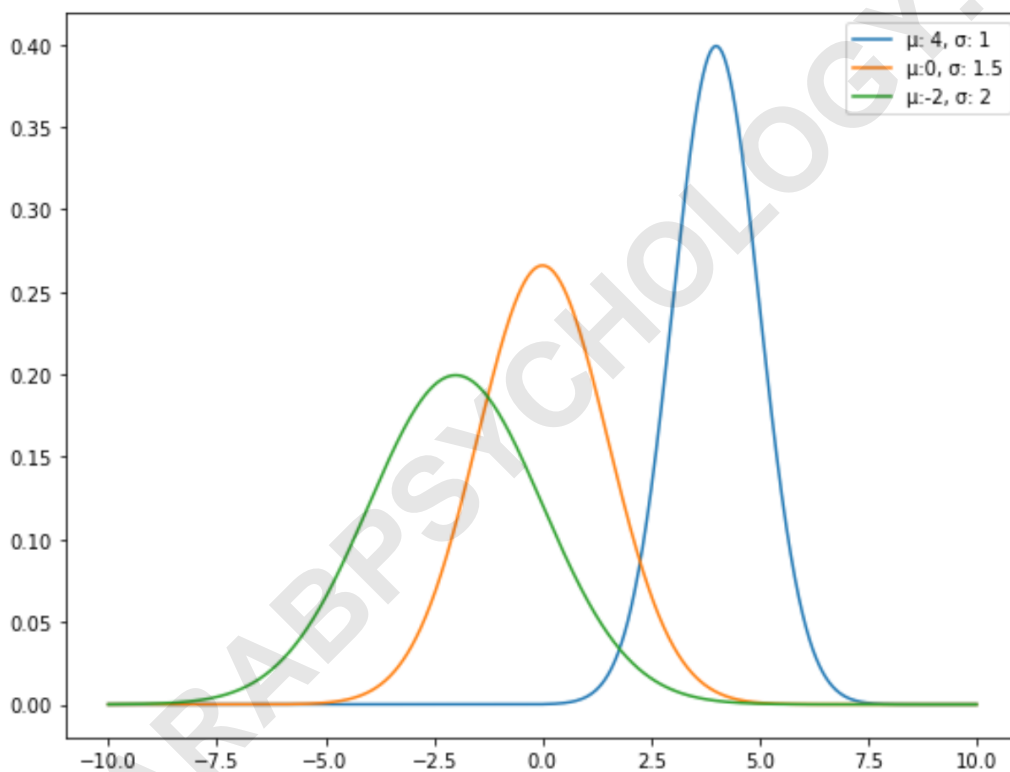
which highlight its distinct shape and symmetry:

The curve is characteristically **Bell-shaped**, smooth, and perfectly symmetrical.

The **Mean** and **Median** are equal; both reside precisely at the peak and center of the distribution.

The mean of the normal distribution determines its **location** on the horizontal axis, and the standard deviation determines its **spread** or variability.

The visual representation below powerfully illustrates how variations in the mean and standard deviation influence the shape and position of the curve. Distributions with larger means are shifted further to the right on the x-axis, whereas distributions with larger standard deviations appear flatter and more spread out, indicating greater variability in the data. Conversely, smaller standard deviations result in a taller, narrower peak, signifying tightly clustered data points around the mean.

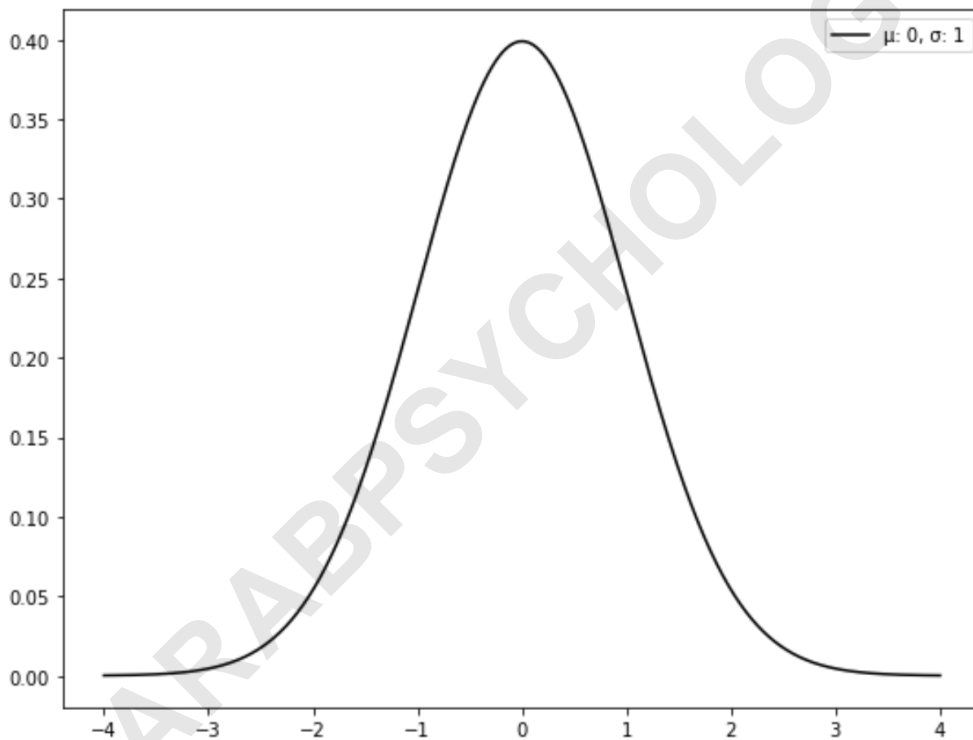


Defining the Parameters of the Standard Normal Distribution (Z-Distribution)

The **Standard Normal Distribution**, symbolized by $N(0, 1)$, holds a unique and crucial position in statistical analysis. It is not just any normal distribution; it is the single, specific normal distribution where the defining parameters are set to their simplest, canonical values: the **mean (μ) is equal to 0** and the **standard deviation (σ) is equal to 1**. This standardization is incredibly powerful because it provides a universal benchmark against which all other normally distributed datasets can be measured and compared.

The necessity of the Standard Normal Distribution arose from the need to simplify complex probability calculations. Prior to its widespread use, calculating probabilities associated with various Normal Distributions required complicated integration. By converting all variables into the standard form, statisticians can utilize a single standardized Z-table (or cumulative distribution function) to find probabilities, thus streamlining the process of determining the likelihood of specific outcomes across any normally distributed dataset.

As mentioned previously, the Standard Normal Distribution is synonymous with the **Z-distribution** because its horizontal axis is calibrated in units of Z-scores. Each Z-score represents how many standard deviations an observation falls above or below the mean of zero. Positive Z-scores indicate values above the mean, while negative Z-scores indicate values below the mean. The illustration below captures the perfectly centralized and standardized nature of this critical distribution, with its peak centered at zero.



The Power of Standardization: Converting to the Standard Normal Distribution

One of the most powerful analytical techniques in statistical modeling is the process of **standardization**, which allows us to transform any observed Normal Distribution, regardless of its mean or standard deviation, into the standard $N(0, 1)$ format. This transformation is achieved by converting every individual data point within the original distribution into its corresponding Z-score.

This process effectively normalizes the dataset, meaning that the transformed distribution will inherently have a mean of 0 and a standard deviation of 1, placing all observations onto a uniform scale.

The ability to convert diverse normal distributions into this single standardized form is vital for comparative analysis. For instance, if a researcher wanted to compare test scores from two different universities--one with a high mean and standard deviation, and another with lower corresponding values--a direct comparison of raw scores would be misleading. By standardizing both sets of scores, the researcher can accurately determine which individual student performed relatively better compared to their own peer group, based purely on their calculated Z-scores.

The mathematical tool used for this crucial conversion is the **Z-score formula**. This formula measures exactly how far, and in what direction (above or below), a raw score is from its distribution's mean, measured in terms of standard deviation units. Understanding and correctly applying this formula is the gateway to utilizing Z-tables and conducting advanced hypothesis testing based on standardized probabilities.

The Z-Score Formula Explained

The conversion of a raw score (x) into a standardized score (z) is governed by a simple yet fundamental equation. This formula is universally applied to any data point within a normally distributed population to calculate its exact position relative to the center of the distribution. The calculation involves two critical steps: first, finding the difference between the raw score and the mean (the deviation), and second, scaling that deviation by dividing it by the standard deviation (the spread).

The formula for calculating the Z-score is presented concisely as:

$$z = (x - \mu) / \sigma$$

Each component of this equation plays a specific role in determining the final standardized score:

x: Individual data value, the raw observation whose standardized position we wish to determine.

μ : Mean of the distribution, serving as the central anchor point for the calculation.

σ : Standard deviation of the distribution, which acts as the unit of measurement for the Z-score.

The resulting Z-score tells us the precise number of standard deviations that x is above or below the mean (μ). A Z-score of +2.0 means the observation is two standard deviations above the mean, while a Z-score of -1.0 indicates that the observation is one standard deviation below the mean. This standardized measure allows for immediate and accurate interpretation of the value's position within the overall distribution.

Practical Application: Calculating Z-Scores for Data Standardization

To solidify the understanding of standardization, consider a practical example involving a dataset that follows a Normal Distribution. Suppose we are analyzing a small sample set where the calculated **mean (μ) is 6** and the calculated **standard deviation (σ) is 2.152**. Our goal is to convert the raw scores (x) into Z-scores (z) to understand the relative position of each observation within the standardized framework.

The original dataset provides the following raw scores:

Data
3
4
5
6
7
8
8
8
8
8
9
9
7
6
6
5
4
4
3
2

To convert these values, we apply the Z-score formula, $z = (x - 6) / 2.152$, for each observation individually. For instance, for the raw score $x=3$, the calculation is $(3 - 6) / 2.152$, resulting in a Z-score of approximately -1.39. This result immediately tells us that the value 3 is situated 1.39 standard deviations below the mean of the original dataset.

Executing this calculation for the entire dataset yields a new, standardized list of values:

Data	Z-Score
3	-1.39
4	-0.93
5	-0.46
6	0.00
7	0.46
8	0.93
8	0.93
8	0.93
8	0.93
8	0.93
9	1.39
9	1.39
7	0.46
6	0.00
6	0.00
5	-0.46
4	-0.93
4	-0.93
3	-1.39
2	-1.86

Upon conversion, the resulting distribution of Z-scores possesses the defining characteristics of the Standard Normal Distribution. Specifically, the mean of this transformed distribution will be exactly zero, and the standard deviation will be exactly one. This confirms that the transformation process has successfully mapped the original data onto the universal Z-distribution scale, thereby simplifying any subsequent probability or statistical calculations and enabling the use of standardized tables.

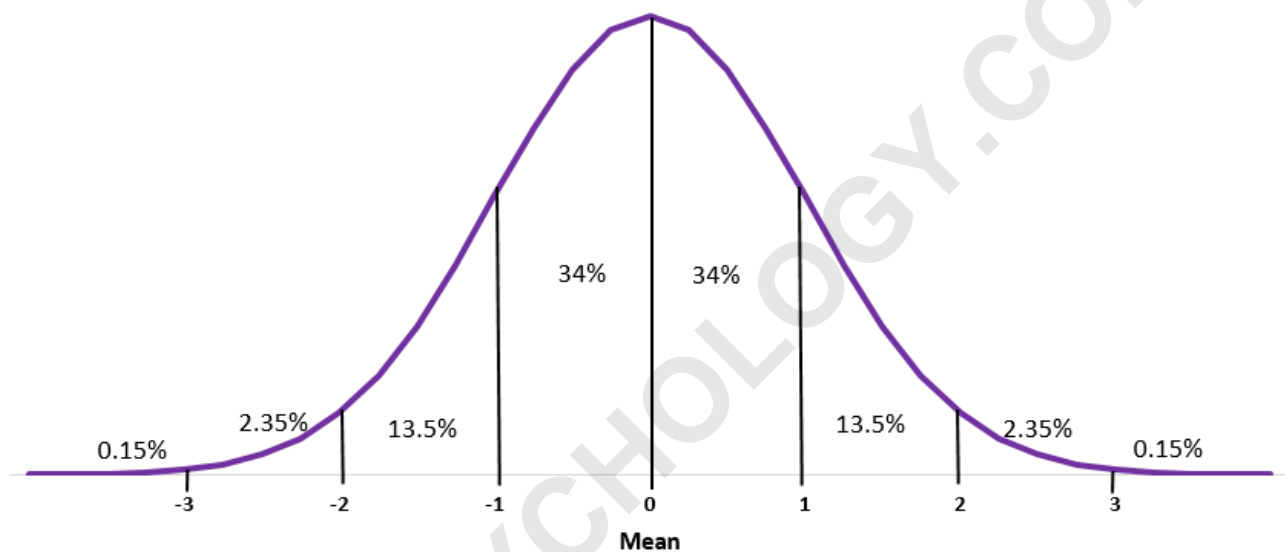
Applying the Standard Normal Distribution: The Empirical Rule (68-95-99.7)

The utility of the Standard Normal Distribution extends far beyond merely calculating Z-scores; it forms the foundation for understanding probability boundaries through the Empirical Rule, sometimes known as the 68-95-99.7 rule. This rule provides a quick, practical estimate of the proportion of data that lies within one, two, or three standard deviations of the mean in any dataset that approximates a Normal Distribution. It allows analysts to rapidly gauge the typicality and spread of observations without resorting to complex integration or Z-tables.

The **Empirical Rule** dictates precise percentages of the data contained within these crucial standard deviation bounds:

About **68%** of data falls within one standard deviation of the mean ($\mu \pm 1\sigma$).
 About **95%** of data falls within two standard deviations of the mean ($\mu \pm 2\sigma$).
 About **99.7%** of data falls within three standard deviations of the mean ($\mu \pm 3\sigma$).

This rule is profoundly important because it ties the measure of spread (standard deviation) directly to the probability of observing a value. A value falling outside three standard deviations ($\pm 3\sigma$) is highly unusual, occurring less than 0.3% of the time. The visual representation below clearly maps these probabilities onto the bell curve, illustrating how data concentrates around the central mean.



Case Study: Using the Empirical Rule for Probability Estimation

To demonstrate the practical application of the Empirical Rule, let us consider a scenario involving plant biology. Suppose the height of plants in a certain garden are normally distributed with a mean height (μ) of 47.4 inches and a standard deviation (σ) of 2.4 inches.

The statistical query is: *According to the Empirical Rule, what percentage of plants are less than 54.6 inches tall?*

The solution involves using the standard deviation steps provided by the Empirical Rule. First, we must confirm that the value 54.6 corresponds to one of the defined standard deviation boundaries (1, 2, or 3 standard deviations away). We calculate the number of standard deviations (the Z-score) using the formula $Z = (x - \mu) / \sigma$. In this case, $Z = (54.6 - 47.4) / 2.4$, which results in a Z-score of 3.0.

Since 54.6 is located precisely three standard deviations above the mean, we can apply the rules of symmetry. We know that 50% of all data values fall below the mean in a normal distribution. We

also know that 99.7% of data falls within three standard deviations (i.e., between $\mu - 3\sigma$ and $\mu + 3\sigma$). Because the curve is symmetrical, the area between the mean and three standard deviations above the mean (from 47.4 to 54.6) must be half of 99.7%, which is 49.85%.

To find the total percentage of plants less than 54.6 inches tall, we sum the proportion below the mean and the proportion between the mean and the target value: $50\% + 49.85\% = 99.85\%$. Thus, **99.85%** of plants are less than 54.6 inches tall, illustrating the efficiency and accuracy of using the Standard Normal Distribution and the Empirical Rule for common probability problems.

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