

How to Use a Chi-Square Distribution Table for Statistical Analysis

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The Foundational Role of the Chi-square Distribution Table

The Chi-square Distribution Table, often simply referred to as the Chi-square table, stands as a cornerstone instrument in the realm of inferential statistics. Its primary function is to facilitate the rigorous process of testing hypotheses concerning the frequency distributions of categorical data. Unlike parametric tests that rely on assumptions about the population parameters (like the mean or standard deviation), the Chi-square test is non-parametric, specifically designed to analyze data that falls into distinct categories, such as survey responses, classifications, or counts. The table provides the necessary benchmark--the critical values--against which researchers can compare their calculated test statistic, ultimately determining whether observed patterns are statistically significant or merely due to random chance. This foundational tool is indispensable for practitioners across scientific disciplines, including social sciences, biology, and market research, providing a standardized method for validating conclusions drawn from frequency data.

The utility of the table derives directly from the properties of the Chi-square distribution itself, which is a specialized probability distribution defined by its degrees of freedom. This distribution is skewed to the right and only includes non-negative values, as the Chi-square statistic (χ^2) is calculated as a sum of squared differences. Understanding the shape and behavior of this distribution for various degrees of freedom is crucial, as the critical value corresponding to a specific significance level shifts dramatically as the degrees of freedom change. The table encapsulates these complex probabilistic relationships into a simple, referenceable format. By navigating the intersection of degrees of freedom and the chosen level of significance, the researcher can instantaneously identify the threshold that separates the region of acceptance for the null hypothesis from the region of rejection, thereby streamlining the decision-making process in statistical inference.

Furthermore, the Chi-square table supports two fundamentally different, yet related, applications: the Chi-square test of independence and the Chi-square goodness-of-fit test. The former is used when examining two categorical variables from a single population to determine if there is a significant association between them (e.g., is gender related to political preference?). The latter is employed to assess whether the observed frequency distribution for a single categorical variable differs significantly from a hypothesized or expected distribution (e.g., do the observed sales figures match the expected uniformity across four quarters?). Despite their differing objectives, both tests culminate in the calculation of a single Chi-square statistic, and both rely entirely upon the same distribution table to interpret that statistic. The robustness and versatility of this tabulated resource ensure consistency and reliability in hypothesis testing across diverse statistical questions involving counts and proportions.

Understanding the Core Components of the Table

When approaching the Chi-square Distribution Table, researchers must first understand the fundamental metrics that structure its layout. The table is systematically organized to provide critical values for the Chi-square statistic (χ^2), which is the quantifiable measure of the discrepancy between the observed frequencies collected during research and the expected frequencies derived from the assumptions of the null hypothesis. The structure of the table reflects the two primary inputs necessary for statistical decision-making: the amount of information available in the sample (represented by degrees of freedom) and the acceptable risk level for making a Type I error (represented by the significance level).

The table's internal structure is typically partitioned based on whether the statistical test being conducted is one-tailed or two-tailed, though in most standard Chi-square applications, the focus is inherently on the upper tail due to the squared nature of the statistic. However, regardless of the test type, the core values listed represent specific percentiles of the Chi-square distribution. For instance, a critical value found at the intersection of 5 degrees of freedom and an alpha level of 0.05 indicates the point on the distribution curve where exactly 5% of the total area under the curve lies to the right. This value, therefore, acts as the cutoff; any calculated Chi-square statistic falling into this 5% tail area leads to the rejection of the null hypothesis, asserting that the observed difference is too large to be attributed to random sampling variation alone.

The following elements are universally present and essential for effective navigation and interpretation of the tabulated data:

The Chi-square Statistic (χ^2): This calculated value quantifies the cumulative difference across all categories between the observed frequencies and expected frequencies. The formula inherently involves squaring the differences, meaning that the Chi-square statistic is always non-negative. A larger calculated χ^2 suggests a greater deviation from the null hypothesis expectations.

Degrees of freedom (df): This row variable dictates the specific shape of the distribution curve being used. It is calculated based on the number of independent categories or groups involved in the test. As degrees of freedom increase, the Chi-square distribution gradually begins to resemble a normal distribution.

Significance level (α): This column variable represents the probability threshold selected by the researcher, typically 0.05 or 0.01. It defines the risk the researcher is willing to take of incorrectly rejecting the true null hypothesis (Type I error rate).

Critical values: These are the numerical values located within the body of the table. They are the benchmark thresholds. If the calculated Chi-square statistic exceeds this value for the specified df and α , the result is deemed statistically significant.

The Concept of Degrees of Freedom (df) in Context

The concept of Degrees of freedom (df) is perhaps the most crucial variable determining the appropriate critical value. In simple terms, degrees of freedom represent the number of values in the final calculation of a statistic that are free to vary. When conducting a Chi-square test, the df is determined not just by the sheer number of observations, but by the constraints imposed by the calculation of the expected frequencies based on the marginal totals. This constraint is fundamental because once the marginal totals are fixed, specifying the frequencies in some cells automatically determines the frequencies in the remaining cells. For example, if you have four categories and you know the total count, specifying the counts for three categories immediately fixes the count for the fourth.

For the Chi-square test of independence, which involves a two-way contingency table with R rows and C columns, the degrees of freedom are calculated using the formula: $df = (R - 1) * (C - 1)$. This mathematical derivation reflects the structural dependence among the cells. Similarly, for the goodness-of-fit test involving K categories, the degrees of freedom are calculated as $df = K - 1 - P$, where P is the number of parameters estimated from the sample data used to determine the expected frequencies. Typically, P is zero, simplifying the calculation to $K - 1$. Accurate determination of the degrees of freedom is non-negotiable, as using the wrong df row in the Chi-square table will inevitably lead to an incorrect critical value and, consequently, a flawed statistical decision.

The relationship between the degrees of freedom and the shape of the Chi-square distribution is profound. As the degrees of freedom increase, the peak of the distribution shifts to the right, and the overall distribution becomes less skewed and more symmetrical, approximating the bell shape of the normal distribution. This change in shape necessitates a corresponding adjustment in the critical value. For a fixed significance level (e.g., $\alpha = 0.05$), a higher degree of freedom will generally require a higher critical value to maintain the same 5% tail probability. Conversely, a low degree of freedom results in a highly skewed distribution where even relatively small critical values are sufficient to reject the null hypothesis (H₀). Therefore, the degrees of freedom row serves as the primary gateway to selecting the correct distributional model for the specific dataset being analyzed.

Interpreting the Significance Level (α) and Probability

The Significance level (α), often displayed in the columns of the Chi-square table, defines the probability threshold used in the decision-making process. It represents the maximum acceptable probability of committing a Type I error--the error of incorrectly rejecting a true null hypothesis. Researchers typically set this value before data collection, most commonly at $\alpha = 0.05$ (5%) or $\alpha = 0.01$ (1%). Choosing a smaller alpha level, such as 0.01, makes the test more stringent,

demanding stronger evidence (a higher calculated Chi-square statistic) before the null hypothesis can be safely dismissed. Conversely, choosing a higher alpha level, such as 0.10, increases the power of the test to detect an effect, but simultaneously raises the risk of a Type I error.

When reading the table, the significance level column heading indicates the area under the curve in the tail(s) of the distribution, corresponding directly to the P-value. The critical value found at the intersection of the chosen df row and the α column demarcates this area. For instance, if the table is structured to show the area in the upper tail, the column labeled 0.05 signifies the critical value (χ^2_{critical}) such that the probability $P(\chi^2 > \chi^2_{\text{critical}})$ equals 0.05. This means that if the null hypothesis were true, one would only expect to calculate a Chi-square statistic greater than or equal to this critical value 5% of the time purely by chance.

In essence, the significance level translates the theoretical probabilistic risk into a practical decision threshold. If the calculated Chi-square statistic from the sample data exceeds the critical value, the associated P-value (the probability of observing such data if H_0 were true) is less than the chosen alpha level. This result is conventionally termed "statistically significant." Conversely, if the calculated statistic falls below the critical value, the P-value is higher than alpha, indicating that the observed data is reasonably likely to occur even if the null hypothesis is correct, leading to the decision to fail to reject H_0 . The clarity and structure of the Chi-square table simplify this complex probabilistic comparison into a straightforward look-up and comparison procedure.

Differentiating One-Tailed and Two-Tailed Tests

While the Chi-square test is primarily used as a non-directional, upper-tailed test due to the nature of the χ^2 calculation (being a sum of squared positive values, differences are always positive and contribute to the upper tail), the concept of one-tailed versus two-tailed hypothesis testing remains crucial for understanding the table's structure, particularly when it is used as a generic resource for assessing probability distributions.

The standard structure of the Chi-square table often focuses exclusively on the upper-tail probability (P-value), especially for goodness-of-fit or independence tests where researchers are interested in detecting deviations from the null hypothesis in any direction (which manifests as a larger-than-expected χ^2 value). In this context, the entire significance level (α) is allocated to the single, upper tail of the distribution. For example, if $\alpha = 0.05$, the critical value corresponds to the 95th percentile of the distribution (the point where 5% of the area is above it). This aligns with the understanding that a rejection of H_0 only occurs when the observed data deviates significantly enough to produce an unusually large test statistic.

However, some complete Chi-square tables also include values corresponding to the lower tail, typically for use in constructing confidence intervals or when a specific statistical application requires assessing the fit across both extremes. The table structure accommodates this

differentiation:

One-tailed: Used when the research hypothesis is directional or when the test statistic naturally focuses on one extreme (the upper extreme for χ^2). For a fixed alpha (e.g., $\alpha = 0.05$), the corresponding critical value is read directly from the column labeled 0.05 (Upper Tail).

Two-tailed: Although less common for categorical Chi-square testing, this structure is important when the alpha level must be split between both tails. If the researcher selects $\alpha = 0.05$, the table might require looking up the critical values corresponding to $\alpha/2 = 0.025$ in the upper tail and $1 - (\alpha/2) = 0.975$ in the lower tail. This ensures that the total rejection region equals 5%.

It is vital for the researcher to confirm the specific convention used by their Chi-square table--whether the column headings represent the probability in the upper tail (most common for χ^2) or the total probability split across both tails. Misinterpreting the column headings regarding tail placement is a frequent source of error in statistical inference, highlighting the need for careful consultation of the table's specific legend or instruction guide before use.

A Step-by-Step Guide to Applying the Chi-square Table

Successfully utilizing the Chi-square table requires a systematic approach, ensuring that the theoretical statistical values are correctly aligned with the empirical data generated from the study. The process begins with the raw data and culminates in a definitive statistical conclusion. The table serves as the indispensable reference point throughout this process, translating raw data comparisons into probability statements.

The practical steps for employing the Chi-square table are as follows:

Calculate your Chi-square statistic (χ^2): This initial and intensive step involves generating the expected frequencies under the assumption that the null hypothesis is true. These expected frequencies are then compared against the observed frequencies using the standardized Chi-square formula: $\chi^2 = \sum \frac{(O_i - E_i)^2}{E_i}$, where O_i represents the observed frequency and E_i represents the expected frequency for each cell or category. This calculation must be meticulous, as any error here invalidates the subsequent critical value comparison.

Determine the Degrees of Freedom (df) and Significance Level (α): Based on the design of the study (number of rows/columns or categories), calculate the appropriate degrees of freedom. Simultaneously, the significance level (α) must be predetermined based on the required level of confidence and the acceptable risk of a Type I error. The chosen df dictates the correct row to use, and α dictates the correct column.

Identify the Appropriate Section (One-tailed/Two-tailed): Confirm whether the test requires a one-tailed or two-tailed interpretation, although, as noted, the standard Chi-square test generally

focuses on the upper tail. If using a table that lists both, ensure the selected column corresponds to the intended tail probability.

Locate the Critical Value: Using the determined degrees of freedom (df) row and the chosen significance level (α) column, locate the corresponding intersection point in the body of the table. This resulting value is the critical value (χ^2_{critical}). This value is the boundary between the non-significant results and the statistically significant results.

Compare your Calculated Chi-square Statistic to the Critical Value: The final step involves a simple comparison between the calculated test statistic ($\chi^2_{\text{calculated}}$) and the table's critical value (χ^2_{critical}). This comparison dictates the decision regarding the null hypothesis.

Drawing Conclusions: Statistical Inference and Decision Making

The entire purpose of calculating the Chi-square statistic and referencing the table converges on the moment of decision-making--the act of statistical inference. This decision is binary: either reject the null hypothesis (H_0) or fail to reject the null hypothesis. The comparison against the critical value provides the objective statistical evidence required to make this determination, moving from raw frequency counts to a meaningful conclusion about the population being studied.

The comparison results in two possible outcomes, each with profound implications for the research findings:

Reject H_0 if your Chi-square statistic is greater than or equal to the critical value. When the calculated $\chi^2_{\text{calculated}}$ lands in the critical region (the far tail of the distribution), it signifies that the observed difference between expected and observed frequencies is so large that it is highly improbable to have occurred if the null hypothesis were true. This outcome indicates a **statistically significant difference or association** between the variables. For example, in a test of independence, rejecting H_0 means there is compelling evidence of a relationship between the two categorical variables.

Fail to reject H_0 if your Chi-square statistic falls below the critical value. If the calculated $\chi^2_{\text{calculated}}$ is smaller than the χ^2_{critical} , it means the observed data is considered consistent with what would be expected under the null hypothesis. The differences observed are likely attributable to random sampling error rather than a genuine effect. This suggests **insufficient evidence for a significant difference or association** at the chosen level of significance. It is crucial to note that failing to reject H_0 does not prove the null hypothesis is true; it merely means the current data lacks the necessary evidence to reject it decisively.

Ultimately, the Chi-square Distribution Table transforms complex probability calculations into an

accessible reference guide, ensuring that statistical decisions are made consistently and objectively based on standardized critical thresholds. It remains a fundamental tool for rigorous analysis of categorical data in scientific investigation.

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df	0.1	0.05	0.025	0.01	0.005	0.001	0.0005	0.0001
1	2.406	3.841	5.024	6.635	7.879	10.828	12.006	15.491
2	1.626	3.000	3.841	5.991	7.378	10.597	11.578	14.443
3	1.495	2.779	3.675	5.841	7.215	10.457	11.476	14.165
4	1.408	2.602	3.541	5.771	7.109	10.342	11.392	13.942
5	1.350	2.501	3.435	5.716	7.043	10.259	11.328	13.788
6	1.310	2.433	3.358	5.671	6.998	10.195	11.277	13.641
7	1.276	2.378	3.291	5.634	6.961	10.141	11.234	13.501
8	1.246	2.328	3.233	5.603	6.930	10.094	11.197	13.368
9	1.220	2.282	3.181	5.578	6.904	10.053	11.165	13.241
10	1.197	2.240	3.135	5.558	6.881	10.018	11.137	13.120
11	1.176	2.201	3.093	5.541	6.860	9.988	11.112	13.004
12	1.157	2.165	3.055	5.526	6.841	9.961	11.089	12.893
13	1.140	2.132	3.021	5.512	6.824	9.937	11.068	12.787
14	1.125	2.101	2.989	5.500	6.809	9.915	11.048	12.685
15	1.111	2.072	2.959	5.489	6.795	9.895	11.030	12.587
16	1.098	2.045	2.931	5.479	6.782	9.876	11.013	12.493
17	1.086	2.020	2.905	5.470	6.770	9.859	11.000	12.402
18	1.075	1.996	2.881	5.462	6.759	9.843	10.988	12.314
19	1.065	1.973	2.858	5.455	6.749	9.829	10.977	12.229
20	1.056	1.951	2.836	5.449	6.740	9.815	10.967	12.146
21	1.047	1.930	2.815	5.443	6.731	9.802	10.958	12.064
22	1.039	1.910	2.795	5.438	6.723	9.790	10.950	11.984
23	1.031	1.891	2.776	5.433	6.715	9.778	10.942	11.905
24	1.024	1.872	2.758	5.429	6.707	9.767	10.935	11.827
25	1.017	1.854	2.741	5.425	6.700	9.756	10.928	11.750
26	1.010	1.837	2.725	5.421	6.693	9.746	10.921	11.674
27	1.004	1.820	2.710	5.417	6.686	9.736	10.915	11.600
28	1.000	1.803	2.695	5.413	6.680	9.726	10.909	11.526
29	0.995	1.787	2.681	5.410	6.674	9.717	10.903	11.453
30	0.991	1.771	2.667	5.406	6.668	9.708	10.897	11.381
31	0.987	1.756	2.654	5.403	6.662	9.700	10.891	11.310
32	0.984	1.741	2.641	5.400	6.656	9.692	10.885	11.240
33	0.981	1.726	2.628	5.397	6.650	9.684	10.879	11.171
34	0.978	1.711	2.616	5.394	6.644	9.676	10.873	11.103
35	0.975	1.696	2.604	5.391	6.638	9.668	10.867	11.035
36	0.972	1.681	2.592	5.388	6.632	9.660	10.861	10.968
37	0.970	1.666	2.580	5.385	6.626	9.652	10.855	10.901
38	0.967	1.651	2.568	5.382	6.620	9.644	10.849	10.835
39	0.965	1.636	2.556	5.379	6.614	9.636	10.843	10.769
40	0.963	1.621	2.544	5.376	6.608	9.628	10.837	10.703
41	0.961	1.606	2.532	5.373	6.602	9.620	10.831	10.637
42	0.959	1.591	2.520	5.370	6.596	9.612	10.825	10.571
43	0.957	1.576	2.508	5.367	6.590	9.604	10.819	10.505
44	0.955	1.561	2.496	5.364	6.584	9.596	10.813	10.439
45	0.953	1.546	2.484	5.361	6.578	9.588	10.807	10.373
46	0.951	1.531	2.472	5.358	6.572	9.580	10.801	10.307
47	0.949	1.516	2.460	5.355	6.566	9.572	10.795	10.241
48	0.947	1.501	2.448	5.352	6.560	9.564	10.789	10.175
49	0.945	1.486	2.436	5.349	6.554	9.556	10.783	10.109
50	0.943	1.471	2.424	5.346	6.548	9.548	10.777	10.043
51	0.941	1.456	2.412	5.343	6.542	9.540	10.771	9.977
52	0.939	1.441	2.400	5.340	6.536	9.532	10.765	9.911
53	0.937	1.426	2.388	5.337	6.530	9.524	10.759	9.845
54	0.935	1.411	2.376	5.334	6.524	9.516	10.753	9.779
55	0.933	1.396	2.364	5.331	6.518	9.508	10.747	9.713
56	0.931	1.381	2.352	5.328	6.512	9.500	10.741	9.647
57	0.929	1.366	2.340	5.325	6.506	9.492	10.735	9.581
58	0.927	1.351	2.328	5.322	6.500	9.484	10.729	9.515
59	0.925	1.336	2.316	5.319	6.494	9.476	10.723	9.449
60	0.923	1.321	2.304	5.316	6.488	9.468	10.717	9.383
61	0.921	1.306	2.292	5.313	6.482	9.460	10.711	9.317
62	0.919	1.291	2.280	5.310	6.476	9.452	10.705	9.251
63	0.917	1.276	2.268	5.307	6.470	9.444	10.700	9.185
64	0.915	1.261	2.256	5.304	6.464	9.436	10.694	9.119
65	0.913	1.246	2.244	5.301	6.458	9.428	10.688	9.053
66	0.911	1.231	2.232	5.298	6.452	9.420	10.682	8.987
67	0.909	1.216	2.220	5.295	6.446	9.412	10.676	8.921
68	0.907	1.201	2.208	5.292	6.440	9.404	10.670	8.855
69	0.905	1.186	2.196	5.289	6.434	9.396	10.664	8.789
70	0.903	1.171	2.184	5.286	6.428	9.388	10.658	8.723
71	0.901	1.156	2.172	5.283	6.422	9.380	10.652	8.657
72	0.899	1.141	2.160	5.280	6.416	9.372	10.646	8.591
73	0.897	1.126	2.148	5.277	6.410	9.364	10.640	8.525
74	0.895	1.111	2.136	5.274	6.404	9.356	10.634	8.459
75	0.893	1.096	2.124	5.271	6.398	9.348	10.628	8.393
76	0.891	1.081	2.112	5.268	6.392	9.340	10.622	8.327
77	0.889	1.066	2.100	5.265	6.386	9.332	10.616	8.261
78	0.887	1.051	2.088	5.262	6.380	9.324	10.610	8.195
79	0.885	1.036	2.076	5.259	6.374	9.316	10.604	8.129
80	0.883	1.021	2.064	5.256	6.368	9.308	10.598	8.063
81	0.881	1.006	2.052	5.253	6.362	9.300	10.592	7.997
82	0.879	991	2.040	5.250	6.356	9.292	10.586	7.931
83	0.877	976	2.028	5.247	6.350	9.284	10.580	7.865
84	0.875	961	2.016	5.244	6.344	9.276	10.574	7.799
85	0.873	946	2.004	5.241	6.338	9.268	10.568	7.733
86	0.871	931	1.992	5.238	6.332	9.260	10.562	7.667
87	0.869	916	1.980	5.235	6.326	9.252	10.556	7.601
88	0.867	901	1.968	5.232	6.320	9.244	10.550	7.535
89	0.865	886	1.956	5.229	6.314	9.236	10.544	7.469
90	0.863	871	1.944	5.226	6.308	9.228	10.538	7.403
91	0.861	856	1.932	5.223	6.302	9.220	10.532	7.337
92	0.859	841	1.920	5.220	6.296	9.212	10.526	7.271
93	0.857	826	1.908	5.217	6.290	9.204	10.520	7.205
94	0.855	811	1.896	5.214	6.284	9.196	10.514	7.139
95	0.853	796	1.884	5.211	6.278	9.188	10.508	7.073
96	0.851	781	1.872	5.208	6.272	9.180	10.502	7.007
97	0.849	766	1.860	5.205	6.266	9.172	10.496	6.941
98	0.847	751	1.848	5.202	6.260	9.164	10.490	6.875
99	0.845	736	1.836	5.199	6.254	9.156	10.484	6.809
100	0.843	721	1.824	5.196	6.248	9.148	10.478	6.743

